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Equivariant cobordism, vector fields, and the Euler characteristic

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Introduction.

Throughout this paper G always denotes a finite group, and a G-manifold means a smooth manifold with smooth G-action. Two n-dimensional closed G-manifolds M and N are *G-cobordant*, if there exists an $(n+1)$-dimensional compact G-manifold L with $\partial L=M+N$, where $+$ denotes the disjoint union. Such a manifold L is called a *G-cobordism* between M and N. If L admits a nonzero G-vector field which is inward normal on M and outward normal on N, then, following Reinhart [7], M and N are called *Reinhart G-cobordant*, and L a *Reinhart G-cobordism* between M and N. The aim of this paper is to obtain a necessary and sufficient condition for the existence of a Reinhart G-cobordism between two given G-cobordant closed G-manifolds.

Given a G-manifold M and a subgroup H of G, M^H denotes the H-fixed point set of M and $M^{=H}$ denotes the union of those components of M^H on which H is the minimal isotropy subgroup. If V is a representation of H containing no direct summand of trivial representation, $M^{(H,V)}$ denotes the union of those components of $M^{=H}$ for which the normal representation is isomorphic to V. Then we will obtain

THEOREM 0.1. *Let M and N be two G-cobordant closed G-manifolds of dimension n. Suppose that n is even and G is of odd order, or that G is of order 2. Then there exists a Reinhart G-cobordism between them if and only if $\chi(M^{(H,V)})=\chi(N^{(H,V)})$ for any pair (H, V) of a subgroup H of G and a representation V of H, where $\chi(\ )$ denotes the Euler characteristic.*

In case H is normal in G, and V is invariant under conjugation, a G-vector bundle $E\to X$ over a G-manifold X is *of type* (H, V) if for any $x\in X$, the isotropy subgroup G_x at x is H, and the fibre E_x over x is isomorphic to V as representations of H. Let $E_1\to X_1$ and $E_2\to X_2$ be G-vector bundles of type (H, V) over k-dimensional closed G-manifolds X_1 and X_2. They are called

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