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The existence of nonexpansive retractions in Banach spaces

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1. Introduction.

In this paper, we consider the existence of nonexpansive retractions concerning nonexpansive mappings. The study of nonexpansive retraction is closely connected to differential equations and geometry of Banach spaces. Especially, it plays an important role in the theory of nonexpansive mappings (cf. Goebel and Reich [5]). In recent years, the existence of nonexpansive retractions has been shown from the mean ergodic theorems for nonexpansive mappings. In [1] Baillon proved the first mean ergodic theorem for nonexpansive mappings: Let C be a closed convex subset of a Hilbert space and let T be a nonexpansive mapping of C into itself. If the set $F(T)$ of fixed points of T is nonempty, then for each $x\in C$, the Cesàro means

$$S_n(x)=\frac{1}{n}\sum_{k=0}^{n-1}T^k x$$

converge weakly to some $y\in F(T)$. This theorem was extended to a uniformly convex Banach space with a Fréchet differentiable norm by Bruck [3], Hirano [6] and Reich [10]. In this case, putting $y=Px$ for each $x\in C$, P is a nonexpansive retraction of C onto $F(T)$ such that $PT=TP=P$ and $Px\in\overline{\mathrm{co}}\{T^n x: n=0, 1, \cdots\}$ for each $x\in C$. In [13], Takahashi proved the existence of such a retraction for an amenable semigroup of nonexpansive mappings in a Hilbert space. Recently Hirano-Takahashi [7] studied this result in a uniformly convex and uniformly smooth Banach space.

In this paper, we prove the existence of such a retraction in a uniformly convex Banach space without additional assumption. Further we find a sequence of means on $N=\{0, 1, \cdots\}$, generalizing the Cesàro means on N, such that the corresponding sequence of mappings converges to such a retraction.

2. Preliminaries.

Let S be a commutative semigroup and let $m(S)$ be the Banach space of all