

The support of global graph links

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Introduction.

In this paper we study global graph links i.e. graph links in three dimensional manifolds, and the main purpose is to answer Tamura's problem below for such links: In [14], [15], Tamura has studied *global knot theory*—the investigation of (codimension two) spheres embedded in manifolds. He called a knot *local* if it is contained in an embedded ball in the ambient manifold and raised in [14] the following

PROBLEM. Give criteria for a knot to be local.

In dimension three, the fundamental group of the exterior determines up to the Poincaré conjecture whether given knot is local or not, since Kneser's conjecture is true (see [21]). However, if one looks for a localness criterion in terms of the homotopy class of the knot, one encounters the following difficulty: Local knots are *inessential* i.e., null homotopic in the ambient manifold, but the converse does not hold. A counterexample in a solid torus is given in Figure 1. By the characterization of S^3 due to Bing [2], any three dimensional manifold admits a knot with irreducible exterior. Thus embedding the above example into a tubular neighborhood of such a knot, we have a counterexample in any manifold. (See Tamura [15] for counterexamples in higher dimensions.)

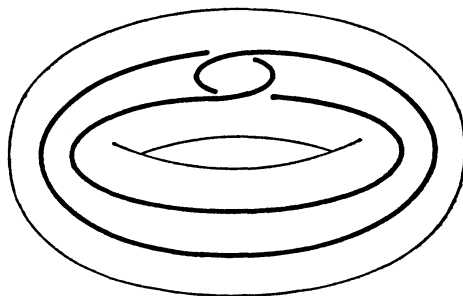


Figure 1.

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