J. Math. Soc. Japan
Vol. 37, No. 4, 1985

Asymptotic behavior of one-dimensional random dynamical systems

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(Received July 12, 1984)

0. Introduction.

Let S be a measurable space and let $\{\tau_s\}_{s\in S}$ be a family of transformations from the unit interval I into itself which are nonsingular with respect to the Lebesgue measure m on I. Given a measure preserving transformation σ and an S-valued random variable ξ on a probability space $(\Omega, \mathcal{F}, P)$, consider a model of a random dynamical system whose time evolution is given by

$$x_{n+1}=\tau_{\xi_{n+1}(\omega)}(x_n) \qquad \text{for} \quad n\geqq 1,$$

where $\xi_n=\xi\circ\sigma^{n-1}$.

Following S. Kakutani [4], we introduce a skew product transformation T on $I\times\Omega$ which is defined by

$$T(x, \omega)=(\tau_{\xi_1(\omega)}x, \sigma\omega) \qquad \text{for} \quad (x, \omega)\in I\times\Omega .$$

Since $\mathrm{proj}_I\circ T^n(x, \omega)=\tau_{\xi_n(\omega)}\tau_{\xi_{n-1}(\omega)}\cdots\tau_{\xi_1(\omega)}x$, we investigate the asymptotic behavior of the dynamical system $(T, m\times P)$ instead of the above random dynamical system. This was done in [3], [6], and [7], in the simplest case when ξ_n's are independent and identically distributed. The aim of this paper is to show that even if $\{\xi_n\}_{n=1}^{\infty}$ is a stationary sequence of dependent random variables, the skew product transformation T has $(m\times P)$-absolutely continuous invariant measures under some mild conditions and T admits various spectral decompositions according to the ergodic property of $\{\xi_n\}_{n=1}^{\infty}$. To do this as in previous paper [6], we introduce the so-called Perron-Frobenius operator $\mathcal{L}$ of T and investigate the asymptotic property of $\mathcal{L}^n\Phi$ for $\Phi\in L^1(m\times P)$. However, instead of estimating $\mathcal{L}^n\Phi$ itself we here estimate $\int_B\mathcal{L}^n\Phi d(m\times P)$ for $B\in \mathcal{B}(I)\times\mathcal{F}$. In Basic Lemma we will give a fundamental inequality on which the whole proof depends heavily (see section 3).

In section 1 we will give the definitions of some basic concepts. The main results are collected in Theorem 2.1. Sections 3 and 4 are devoted to the proof of Theorem 2.1. In section 5 we will give an application of the theorem to random stochastic matrices.