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Torus fibrations over the 2-sphere with the simplest singular fibers

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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§ 1. Introduction.

By a *torus fibration* $f: M \to B$ is roughly meant a certain singular fibration of an oriented smooth 4-manifold M over an oriented surface B with general fiber the 2-torus. (For a precise definition, see Definition 2.1.) Some special types of such fibrations have been studied by Thornton [**11**] and Zieschang [**13**] as a generalization of Seifert fibered spaces (into higher dimensions not necessarily 4 in their articles), and other special types by Harer [**1**] and Moishezon [**9**] as a smooth analog of Lefschetz' pencils or Kodaira's elliptic fiber spaces [**4**]. (General fibers of Harer's pencils need not be tori.) The author gave a general formulation of torus fibrations [**6**].

Among the possible types of singular fibers that torus fibrations can admit, the simplest one would be of type I_1^+ or I_1^-. A singular fiber of type I_1^+ (resp. I_1^-) consists of a smoothly immersed 2-sphere with a single transverse self-intersection of sign $+1$ (resp. -1).

In this paper we will deal with torus fibrations over the 2-sphere whose singular fibers are of type I_1^+ or I_1^-. Our goal will be to classify the (not necessarily fiber preserving) diffeomorphism types of the total spaces of such torus fibrations. The following is our main result.

THEOREM 1.1. *Let $f: M \to S^2$ be a torus fibration over the 2-sphere each of whose singular fibers is of type I_1^+ or I_1^-. Suppose that the signature of M is not zero. Then M is 1-connected, and the diffeomorphism type of M is determined by the euler number $e(M)$ and the signature $\sigma(M)$.*

REMARK. Assume that each singular fiber of a torus fibration $f: M \to S^2$ is of type I_1^+ or I_1^-, and that there are k_+ singular fibers of type I_1^+ and k_- singular fibers of type I_1^-. Then $e(M)$ and $\sigma(M)$ are given by $e(M)=k_+ + k_-$, $\sigma(M)$

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