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Real hypersurfaces of a complex hyperbolic space

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1. Introduction.

During the last years the study of real hypersurfaces of Kaehlerian manifolds has been an important subject in geometry of submanifolds, especially when the ambient space is a complex space form.

One of the first results in this way (see [12]) was to state that any real hypersurface M of a complex space form $\bar{M}(c)$ with holomorphic sectional curvature $c\neq 0$ is not totally umbilical. This is a direct consequence of classical Codazzi's equation for such a hypersurface. From that equation, also one can deduce that there does not exist real hypersurfaces M of $\bar{M}(c)$, $c\neq 0$, with parallel second fundamental form H. So, it seems interesting to describe and characterize real hypersurfaces of $\bar{M}(c)$, $c\neq 0$, with a few principal curvatures or with derivative ∇H of the second fundamental form of short length. These problems have been solved, in the case $c>0$, in [2], [6], [10], [11] and other works.

On the other hand, Kon, in [5], stated that there are no Einstein real hypersurfaces in $\bar{M}(c)$, $c>0$, and he studied a less restrictive condition for the Ricci tensor of these hypersurfaces: the pseudo-Einstein condition (see also [6]). In fact, he classified the pseudo-Einstein real hypersurfaces of the complex projective space using Takagi's works [10] and [11].

Finally, Cecil and Ryan generalized in [2] some results of [10] and [5]. They described in terms of tubes over complex submanifolds the real hypersurfaces of the complex projective space which appear in the literature.

Now we are interested in these problems when $c<0$, that is, when $\bar{M}(c)$ is the complex hyperbolic space CH^m (for convenience we will assume $c=-4$). So, A. Romero and the author have classified in [7] all complete real hypersurfaces of CH^m which admit a S^1-principal bundle which is a parallel Lorentzian hypersurface of the anti-De Sitter space H_1^{2m+1}. These real hypersurfaces have the least length for ∇H as we will show in a forthcoming paper.

In this paper we construct some examples of real hypersurfaces of CH^m (Section 6) and we give a complete classification of the real hypersurfaces of CH^m with at most two principal curvatures at each point. In this classification we