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Exceptional manifolds for generalized Schoenflies theorem

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In Tamura [2] the generalized Schoenflies theorem for spheres was proved. The statement is as follows:

THEOREM. *Let M be a connected orientable smooth n-manifold satisfying one of the following conditions:*

i) *M is noncompact or with nonempty boundary,*

ii) *M has a non-zero j-th Betti number for some $j \neq 0$, n,*

iii) *The fundamental group of M is an infinite group,*

iv) *M is a homology sphere.*

Then every inessential $(n-1)$-sphere embedded in M bounds an embedded n-disk.

Also in [3] the generalized Schoenflies theorem for $S^p \times S^q$ was proved:

THEOREM. *Let M be a manifold as in the above theorem. Let $p+q=n-1$. Then every inessential $S^p \times S^q$ embedded in M bounds an embedded $D^{p+1} \times S^q$ or an embedded $S^p \times D^{q+1}$.*

Following Tamura [2], a manifold in which some inessential embedded sphere (resp. $S^p \times S^q$) does not bounds an embedded disk (resp. $D^{p+1} \times S^q$ or $S^p \times D^{q+1}$) is said to be *exceptional*. In this paper we prove that exceptionality for sphere is equivalent to that for $S^p \times S^q$ in most cases. Theorem 1 below also shows that the Schoenflies theorem for spheres implies the Schoenflies theorem for $S^p \times S^q$.

Throughout this paper we work in PL category. Although [2], [3] deal with smooth manifolds, the PL versions of the theorems can be proved in the same way. All manifolds are assumed to be connected and orientable. S^m denotes the m-sphere, and D^m denotes the m-disk centered at 0. "Link" denotes the linking number.

THEOREM 1. *If $1<q<p-1$, every $(p+q+1)$-dimensional manifold which is exceptional for $S^p \times S^q$ is exceptional for S^{p+q}.*

PROOF OF THEOREM 1. Let M be a $(p+q+1)$-dimensional manifold which is not exceptional for S^{p+q}; i.e. every inessential $(p+q)$-sphere embedded in M bounds an embedded $(p+q+1)$-disk. We will prove that every inessential $S^p \times S^q$