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The automorphism group of Leech lattice and elliptic modular functions

Dedicated to Professor Hirosi Nagao on his 60th birthday

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Introduction.

As usual, we denote by $\cdot 0$ the automorphism group of Leech lattice which is an even unimodular lattice in 24-dimensional Euclidean space [**1**]. So $\cdot 0$ has a natural 24-dimensional representation ρ_0 over the rational number field. In this paper, Frame shapes of conjugacy classes of $\cdot 0$ with respect to ρ_0, the list of which is given in Table I of Appendix, will play a central role. For the definition of Frame shape, see § 1.2.

Let $\mathfrak{F}$ be the set of all elliptic modular functions $f(z)$ satisfying the following conditions:

(1) $f(z)$ is a modular function with respect to a discrete subgroup Γ of $SL(2, \boldsymbol{R})$ containing $\Gamma_0(N)$ for some integer N (i.e. $f\left(\frac{az+b}{cz+d}\right)=f(z)$ for any $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$ and meromorphic on the upper half plane and at all cusps of Γ),

(2) the genus of Γ is zero and $f(z)$ is a generator of a function field for Γ,

(3) $f(z)$ has a Fourier expansion of the form $f(z)=1/q+\sum_{n=0}^{\infty} a_n q^n$ $(q=e^{2\pi i z})$.

Now the main result of this paper is to show that *various "transformations"* (cf. § 1.1) *of Frame shapes of* $\cdot 0$ *yield functions of* $\mathfrak{F}$ (Th. 3.2, 3.4, 3.5 and Table II~IV in Appendix). Furthermore, an application of this result is as follows: Let G be a finite group which has a d-dimensional representation ρ over the rational number field where d is a divisor of 24. For each of such many (not all) pairs (G, ρ), we can construct a mapping from G to $\mathfrak{F}$

$$G \ni \sigma \longmapsto j_\sigma(z) \in \mathfrak{F}$$

such that all coefficients $a_k(\sigma)$ $(k \geqq 1)$ of a Fourier expansion $j_\sigma(z)=1/q+\sum_{k=0}^{\infty} a_k(\sigma) q^k$ are generalized characters of G (Th. 4.6, 4.8 and 4.10). Such a mapping is called a *moonshine* of G. A moonshine of Fischer-Griess's Monster is constructed in a remarkable paper of Conway-Norton [**2**] and other examples of moonshines can be found in Queen [**10**] and Koike [**4**]. Constructions of moon-