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On the value distribution of meromorphic mappings of covering spaces over C^m into algebraic varieties

Dedicated to Professor M. Ozawa on his 60th birthday

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Introduction.

The purpose of this paper is to study the meromorphic mappings $f: X \to V$ of a finite analytic (ramified) covering space X over the m-dimensional complex vector space $\boldsymbol{C}^m$ with projection $\pi: X \to \boldsymbol{C}^m$ into a complex projective manifold V of dimension n from the view point of the Nevanlinna theory; especially, we are interested in inequalities of the second main theorem type. In the case where $m=1$ and V is the 1-dimensional complex projective space $\boldsymbol{P}^1(\boldsymbol{C})$, Selberg [17] proved the first and the second main theorems for $f: X \to \boldsymbol{P}^1(\boldsymbol{C})$. In the case where $\dim X \geqq \dim V = \operatorname{rank} f (= \sup\{\dim X - \dim_x f^{-1}(f(x)); x \in X\}) \geqq 1$, we proved the second main theorem and defect relations for f and divisors on V, generalizing the above results of Selberg and the Carlson-Griffiths-King theory [2] and [5] (see [10], [11] and [19]).

Here we deal with the case where $\operatorname{rank} f$ does not necessarily equal $\dim V$. Stoll [20] obtained the Ahlfors-Weyl theory for linearly non-degenerate meromorphic mappings from a parabolic manifold into $\boldsymbol{P}^n(\boldsymbol{C})$ which applies to the case of $f: X \to \boldsymbol{P}^n(\boldsymbol{C})$ (see [20, Theorem 11.8]). When X is an affine algebraic curve (or the domain of f may be the punctured disc Δ^* in $\boldsymbol{C}$), we proved an inequality of the second main theorem type for $f: X \to V$ in terms of logarithmic 1-forms along the given divisors on V (see [12], [13] and [14]), and applied it to obtain a generalization of big Picard's theorem (see [14]). In the present paper we extend this inequality to the case where X is a finite analytic covering space over $\boldsymbol{C}^m$, and give an application.

Let D be an effective reduced divisor on V such that the closed image of the quasi-Albanese mapping $\alpha: V - D \to A_{V-D}$ is of dimension n and of general type (cf. [6] and [7]). Here we identify D with its support. Let $f: X \to V$ be a meromorphic mapping. We say that f is degenerate with respect to the complete linear system $|L|$ of a holomorphic line bundle L over V if $f(X) \subset \operatorname{Supp} E$ for some $E \in |L|$, where $\operatorname{Supp} E$ denotes the support of the divisor E; otherwise, f is said to be non-degenerate with respect to $|L|$. Assume that $f(X) \not\subset D$ and