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On integral transformations associated with a certain Lagrangian
— as a prototype of quantization

Dedicated to the memory of the late Professor H. Kumano-go

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Introduction.

Let M be a finite dimensional manifold and let $L(\gamma, \dot{\gamma})$ be a function on the tangent bundle TM. Our aim is to construct a C^0-*semi* group of bounded linear operators $\boldsymbol{H}_t^\lambda(L)$ associated with $L(\gamma, \dot{\gamma})$ and its infinitesimal generator $\boldsymbol{A}^\lambda(L)$ on the intrinsic Hilbert space $\mathcal{H}(M)$ (see §5 for its definition), where $t\in\boldsymbol{R}_+$ and λ is a positive parameter.

As the above problem is too vague to consider, we restrict ourselves to the following case which seems rather typical.

(M) M is a smooth, simply-connected and connected d-dimensional manifold.

(L. I) $L(\gamma, \dot{\gamma})$ is represented by

$$L(\gamma, \dot{\gamma})=L^0(\gamma, \dot{\gamma})-V(\gamma), \qquad L^0(\gamma, \dot{\gamma})=(1/2)g_{ij}(\gamma)\dot{\gamma}^i\dot{\gamma}^j \tag{1}$$

for $(\gamma, \dot{\gamma})\in TM$. (Hereafter, we use Einstein's convention to contract indices.)

Moreover,

(L. II) $ds^2=g_{ij}(x)dx^idx^j$ defines a complete Riemannian metric on M.

(In the following, for such $g_{ij}(x)$, we associate quantities in Riemannian geometry as are used usually.)

(L. III) There exists a constant $k\geqq 0$ such that for any 2-plane π, the sectional curvature K_π satisfies $-k^2\leqq K_\pi\leqq 0$.

(L. IV) Denote by $R_{ijk}{}^h(x)$ the component of curvature tensor $R(\cdot\,,\cdot)$. Then, there exists a constant C_0 such that

$$|\nabla^\alpha R_{ijk}{}^h(x)|\leqq C_0 \qquad \text{for} \quad 0\leqq|\alpha|\leqq 3\,,$$