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The mod 2 semicharacteristic and groups acting freely on manifolds

To the memory of Dr. Takehiko Miyata

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Introduction.

In [8, 9] the author established the equivariant point theorem for a continuous map between manifolds with free involution, and applied it to re-prove the theorems of Milnor [6], Lee [4] and Stong [11] on groups acting freely on manifolds. In this paper, we shall show that the following theorems on group actions are also proved easily by making use of the equivariant point theorem.

THEOREM 1. *Suppose that a group G acts freely on a closed $(2n+1)$-dimensional manifold M. Let $\sigma, \tau \in G$ be elements such that $\tau^2=1$ and $\sigma\tau \neq \tau\sigma$. Then the trace of*

$$(\sigma\tau\sigma^{-1}\tau^{-1})^* : \bigoplus_{k\leq n} H^k(M; \boldsymbol{Z}_2) \longrightarrow \bigoplus_{k\leq n} H^k(M, \boldsymbol{Z}_2)$$

is zero.

This is a generalization of Theorem 1 in Montgomery-Yang [7]; they deal with the case G is a dihedral group.

We denote by $k_2(M)$ the mod 2 semicharacteristic of a closed $(2n+1)$-dimensional manifold M:

$$k_2(M) = \sum_{k\leq n} \dim H^k(M; \boldsymbol{Z}_2) \qquad \text{mod } 2.$$

THEOREM 2. *If a 2-group G acts freely on a closed orientable $(4n+1)$-dimensional manifold M with $k_2(M) \neq 0$, then G is cyclic.*

We denote by $k_0(M)$ the rational semicharacteristic of a closed orientable $(2n+1)$-dimensional manifold, i.e.

$$k_0(M) = \sum_{k\leq n} \dim H^k(M; \boldsymbol{Q}) \qquad \text{mod } 2.$$

If n is even and M admits a free action of $\boldsymbol{Z}_2$, then we have $k_0(M)=k_2(M)$ by the formula of Lusztig-Milnor-Peterson [5]. Therefore Theorem 2 coincides with

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