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# Automorphism groups of multilinear mappings

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## 1. Introduction.

The relation between the nonsingularity of a multilinear mapping and the finiteness of its automorphism group was recently studied by the second author [2]. In particular, it was shown ([2, Theorem A]) that the nonsingularity implies the finiteness under some restriction on the characteristic of the underlying field. In this paper we shall prove the same result without this restriction.

If $V$ is a vector space over a field, a multilinear mapping

$$\theta : V\times \overset{r}{\cdots\cdots} \times V \longrightarrow V$$

from the direct product of $r$ copies of $V$ into $V$ itself is called simply a multilinear mapping of degree $r$ on $V$. The subgroup $\mathrm{Aut}(\theta)$ of the general linear group $\mathrm{GL}(V)$ is defined by

$$\mathrm{Aut}(\theta)=\{\varphi\in\mathrm{GL}(V) \mid \theta(x_1, x_2, x_3, \cdots, x_r)^{\varphi}=\theta(x_1^{\varphi}, x_2^{\varphi}, x_3^{\varphi}, \cdots, x_r^{\varphi}) \quad \text{for all} \quad x_1, x_2, x_3, \cdots, x_r\in V\}.$$

We say that $\theta$ is nonsingular, if $\theta(x, x, x, \cdots, x)\neq 0$ for all $0\neq x\in V$.

Our main result is:

THEOREM A. *Let $\theta$ be a nonsingular multilinear mapping of degree $r\geqq 2$ on a vector space $V$ of dimension $n$ over an algebraically closed field $F$ of characteristic $p>0$. Then* $\mathrm{Aut}(\theta)$ *is a finite group.*

Theorem A can be derived from the following Theorem B.

THEOREM B. *Let $F, p, V, n, \theta, r$ be as in Theorem* A. *Then for every unipotent subgroup $Q$ of* $\mathrm{Aut}(\theta)$,

$$|Q|\leqq p^{\sum_{i\geqq 1}[n/p^i]},$$

*where $[\ ]$ denotes the greatest integer not exceeding the number inside.*

That Theorem B implies Theorem A follows from the following two propositions which appear as Propositions 1 and 6 in [2]. (In these two propositions, the characteristic is arbitrary. In Proposition D, the field need not be algebraically closed.)