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Projective plane curves and the automorphism groups of their complements

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1. Introduction.

Let C be an irreducible algebraic curve of degree d on $\boldsymbol{P}^2=\boldsymbol{P}^2(\boldsymbol{C})$ and put $V=\boldsymbol{P}^2\setminus C$. Let $\mathcal{G}$ be the automorphism group of the algebraic surface V and $\mathcal{L}$ the linear part of $\mathcal{G}$, i.e., $\mathcal{L}=\{T\in \mathrm{Aut}(\boldsymbol{P}^2)\mid T(C)=C\}$. If $d=1$, then $\mathcal{G}$ is generated by linear transformations and de Jonquières transformations of V (Nagata [**5**]); if $d=2$, then generators of the similar kind have been found by Gizatullin and Danilov [**2**]. In this paper we shall study the structure of $\mathcal{G}$ and at the same time the property of C in the case when $d\geqq 3$.

We shall use the following notations in addition to the above ones. Let (X, Y, Z) be a set of homogeneous coordinates on $\boldsymbol{P}^2$ and put $x=X/Z$ and $y=Y/Z$. Usually we do not treat the line $Z=0$, so we say that for an irreducible polynomial f, the curve $Z^d f(X/Z, Y/Z)=0$ is defined by f, where $d=\deg f$. Especially we denote by Δ [resp. Δ_e] the curve defined by $xy-x^3-y^3$ [resp. y^e-x^d, where $(e, d)=1$ and $1\leqq e\leqq d-2$]. Let M be the number of the singular points $\{P_1, \cdots, P_M\}$ of C and $\mu:\tilde{C}\to C$ the normalization of C. Then let N denote the number of elements of $\mu^{-1}(\{P_1, \cdots, P_M\})$ and g the genus of $\tilde{C}$. In case $N=1$, let $(e_1, \cdots, e_p)$ be the sequence of the multiplicities of all successive infinitely near singular points of P_1, and put

$$R=d^2-\sum_{i=1}^{p} e_i^2-e_p+1\,.$$

Let $\boldsymbol{G}_a$ and $\boldsymbol{G}_m$ be the additive and the multiplicative groups respectively.

First we shall prove the following with the help of the Plücker relations.

PROPOSITION 1. *Suppose that $d\geqq 3$. Then the following three conditions are equivalent.*

(1) *The order of $\mathcal{L}$ is infinite.*
(2) *The linear part $\mathcal{L}$ is isomorphic to $\boldsymbol{G}_m$.*
(3) *The curve C is projectively equivalent to Δ_e.*

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