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The first eigenvalue of Laplacians on minimal surfaces in S^3

Dedicated to Professor Naomi Mitsutsuka on his 60th birthday

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1. Introduction.

There are many complete surfaces with constant mean curvature in the Euclidean 3-space $\boldsymbol{R}^3$ and in the hyperbolic 3-space $\boldsymbol{H}^3$ (see [2], [4]). But in the Euclidean 3-sphere S^3 there have been few results on such surfaces except umbilic ones and flat tori (cf. [5]).

In this paper, we shall construct a one-parameter family of complete, rotational surfaces in S^3 with constant mean curvature, including a flat torus as an initial one. In particular, there is a one-parameter family of complete, rotational, minimal surfaces in S^3, including the Clifford torus. And we shall show that none of closed, rotational, minimal surfaces in S^3 is embedded and the first eigenvalues of some ones relative to the Laplacian are smaller than *two* except for the Clifford torus.

2. Preliminaries.

In this section, we shall review rotational surfaces in S^3. At first, we note that S^3 is realized as a hypersurface of the Euclidean 4-space $\boldsymbol{R}^4$:

$$S^3=\{(x_1, \cdots, x_4)\in \boldsymbol{R}^4 ; \sum_j x_j^2=1\}.$$

In what follows, we denote by $S^2(c)$ the Euclidean 2-sphere of constant Gaussian curvature c (or equivalently, the 2-sphere in $\boldsymbol{R}^3$ of radius $1/\sqrt{c}$), and by $S^1(r)$ the circle in $\boldsymbol{R}^2$ of radius r. And we put $S^1=S^1(1)$ and $\boldsymbol{R}=S^1(\infty)$ for convenience's sake. We note that $S^1(r)\equiv \boldsymbol{R}/2\pi r\boldsymbol{Z}$ for a positive number r, where $\boldsymbol{Z}$ is the set of all integers.

Up to an isometry of S^3, an umbilic surface and a flat torus in S^3 are represented as follows. For each real number H, the isometric embedding $f: S^2(H^2+1)\rightarrow S^3$, $f(x, y, z)=(x, y, z, H/\sqrt{(H^2+1)})$ of $S^2(H^2+1)$ into S^3 defines an umbilic surface $M^2(H)$ in S^3 with constant mean curvature H, and for $a=\sqrt{[\{1-H/\sqrt{(H^2+1)}\}/2]}$ and $b=\sqrt{(1-a^2)}$, the isometric embedding $f: S^1(a)\times S^1(b)\rightarrow S^3$, $f((x, y), (u, v))=(x, y, u, v)$ of $S^1(a)\times S^1(b)$ into S^3 defines a flat torus