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On cyclotomic units connected with p-adic characters

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§1. Introduction.

Let p be an odd prime and let K be an abelian number field of degree prime to p which contains a primitive p-th root of unity. We denote by η_{ϕ} a ϕ-relative cyclotomic unit in the sense of Gras [2], where ϕ is a non-trivial even p-adic character of the Galois group of K over the rationals. Gras has given some congruences concerning η_{ϕ} and Bernoulli numbers associated with the reflection $\bar{\phi}$ of ϕ. Let $A(\phi)$, $A(\bar{\phi})$ be p-subgroups of the ideal class group of K corresponding to ϕ, $\bar{\phi}$ respectively. A close relation between $A(\phi)$ and $A(\bar{\phi})$ was stated by Leopoldt [5]. Recently Wiles [8] proved that if K is the p-th cyclotomic field and η_{ϕ} is a p-th power in K then $A(\phi)$ is non-trivial.

In this paper we shall give a relation between η_{ϕ} and $A(\bar{\phi})$. Namely we state a necessary and sufficient condition for η_{ϕ} to be a p-th power in K in terms of the ideals representing classes in $A(\bar{\phi})$. In the case that K is the p-th cyclotomic field, Iwasawa has shown the above result applying a theorem of Artin-Hasse concerning power residue symbols (cf. [3], Lemma 3). On the other hand our proof is essentially based on the prime factorization of certain Jacobi sums.

§2. Notation and results.

Throughout this paper we denote by p an odd prime and by $\boldsymbol{Z}$, $\boldsymbol{Z}_p$, $\boldsymbol{Q}$, and $\boldsymbol{Q}_p$ the ring of rational integers, the ring of p-adic integers, the field of rational numbers, and the field of p-adic numbers respectively. Further it is assumed that all integers and all algebraic number fields are contained in an algebraic closure $\bar{\boldsymbol{Q}}_p$ of $\boldsymbol{Q}_p$. For a rational integer $m>0$ let ζ_m be a primitive m-th root of unity.

Let K be an abelian number field and let χ be a character of the Galois group $\mathrm{Gal}(K/\boldsymbol{Q})$. By $g(\chi)$ we always mean the order of χ. Let K_{χ} be the fixed field of the kernel of χ. Then K_{χ} is a cyclic extension of $\boldsymbol{Q}$ of degree $g(\chi)$.

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