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L^p-estimates of solutions of some nonlinear degenerate diffusion equations

Dedicated to Professor Sigeru Mizohata
on his sixtieth birthday

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0. Introduction.

The object of this paper is to show the existence, uniqueness and L^p estimates (including $p=\infty$) of global solutions of some nonlinear degenerate diffusion equations.

The first problem we are concerned with is the following initial-boundary value problem of the perturbed porous medium equation;

$$(\mathrm{P}_1)\quad \begin{aligned} &\frac{\partial}{\partial t}u-\Delta u^{m+1}+f(x, t, u)=0 \quad \text{in} \quad \Omega\times(0, \infty) \\ &u(x, 0)=u_0(\geqq 0), \quad u|_{\partial\Omega}=0 \quad \text{and} \quad u\geqq 0 \end{aligned}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$ (C^3 class is sufficient), m is a positive constant and $f(x, t, u)$ is a function satisfying;

ASSUMPTION 1. (i) $f(x, t, u)$ is locally Hölder continuous in $\bar{\Omega}\times R^+\times R^+$ ($R^+=[0, \infty)$) and locally Lipschitz continuous with respect to u uniformly in (x, t).

(ii) $f(x, t, u)\geqq -C_0u^{1+\alpha}$ on $\bar{\Omega}\times R^+\times R^+$ for some $C_0>0$ and $\alpha\geqq 0$.

It should be noted that the theory of nonlinear semi-groups does not apply to (P_1) for the existence of global solution since we do not assume that $f(x, t, u)$ is monotone with respect to u.

To treat the problem (P_1) it is convenient to compare it with the problem;

$$(\mathrm{P}_2)\quad \begin{aligned} &\frac{\partial}{\partial t}u-\Delta u^{m+1}-C_0u^{1+\alpha}=0 \quad \text{in} \quad \Omega\times(0, \infty) \\ &u(x, 0)=u_0(\geqq 0), \quad u|_{\partial\Omega}=0 \quad \text{and} \quad u\geqq 0. \end{aligned}$$

Recently in [**13**] we have discussed the existence, nonexistence and some

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