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Groundedness property and accessibility of ordinal diagrams

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§ 0. Introduction.

The theory of ordinal diagrams as well as its applications and generalizations has been worked out over the years by G. Takeuti and the present author. (See [1]~[6]; all the earlier results in this line are included in [2].) The system of ordinal diagrams was invented by Takeuti as a means for the consistency proofs; that is, the consistency of a subsystem of analysis is reduced to the accessibility of ordinal diagrams. It is therefore of primary importance that we establish an accessibility proof of ordinal diagrams in its strict sense. Such attempts have been made in [2], [7] and [8], but none of them is entirely satisfactory from our standpoint. (We refer the reader to Sections 11 and 26 of [2] for a detailed discussion on the constructive standpoint.)

Let $\mathcal{J}=(J, <)$ be a concretely given linearly ordered structure. An accessibility proof of $\mathcal{J}$ consists in presenting a "concrete" method to establish that there be no infinite $<$-decreasing sequence from J, and J is said to be $<$-accessible if there is such an accessibility proof.

Here in this article the author presents a more constructive accessibility proof of ordinal diagrams.

Let I and A be two accessible sets and let $\boldsymbol{O}(I, A)$ be the system of ordinal diagrams based on I and A. Takeuti originally proved the accessibility of $\boldsymbol{O}(I, A)$ by making use of a subset of it, which he named F_i, for each i an element of $I\cup\{\infty\}$ (Section 26 of [2]). He later became declined to accept F_i as a concrete object, and he and the present author proposed another version in [7], in which the theory of fundamental sequences in $\boldsymbol{O}(I, A, S)$ (an extended system of ordinal diagrams) for some S and the notion of strong accessibility stand essential. Since the construction of fundamental sequences is finitary (see [3]), the problem of this approach can be pinned down to the notion of strong accessibility, which is defined in terms of arbitrary well-ordered sets.

Takeuti has recently revisited the theme of accessibility in the appendix of [8], resorting to the sets F_i's once again. In [8] he defines F_i for each i by the condition that, for an α to be in F_i, "there be a method" to show that every