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The normality of Σ-products and the perfect κ-normality of Cartesian products

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§ 0. Introduction.

Corson [**3**] introduced the concept of Σ-products, which are quite important subspaces of Cartesian products of topological spaces. He studied there the normality of Σ-products. On the other hand, Blair [**2**], Ščepin [**16**] and Terada [**19**] independently introduced the concept of perfect κ-normality (or Oz) which is analogous to that of normality. The former two studied there when Cartesian products of topological spaces are perfectly κ-normal. In these connections, the following two results (I) and (II) seem to be most remarkable:

(I) A Σ-product of metric spaces is (collectionwise) normal.

(II) A Cartesian product of metric spaces is perfectly κ-normal.

The former was proved by Gul'ko [**4**] and Rudin [**9**]. The latter was given by Ščepin [**16**]. Subsequently, Kombarov [**8**] obtained a nice extension of (I) as follows:

(III) For a Σ-product Σ of paracompact p-spaces, (a) Σ is normal, (b) Σ is collectionwise normal and (c) Σ has countable tightness are equivalent.

As another generalized metric spaces, Okuyama [**13**] introduced the concept of σ-spaces. Subsequently, Nagami [**11**] introduced the class of Σ-spaces which contains both ones of σ-spaces and paracompact p-spaces. These generalized metric spaces play important roles in this paper.

Recently, the author [**21**] has proved that for a Σ-product Σ of paracompact Σ-spaces the implication (c)$\Rightarrow$(b) in (III) is true. The first purpose of this paper is to prove that for such a Σ-product Σ the implication (a)$\Leftrightarrow$(b) is true. We also discuss the countable paracompactness of Σ-products. The second purpose of this paper is to obtain an extension of (II) for a Cartesian product of paracompact σ-spaces, the form of which is resemble to that of (c)$\Rightarrow$(a) in (III). In process of proving this result, we consider the union of $\aleph_0$-cubes in a Cartesian product of σ-spaces. This is closely related to a certain question of R. Pol and E. Pol [**14**] though it has been already solved by Klebanov [**5**].

All spaces considered here are assumed to be Hausdorff. The letters n, i, j,