J. Math. Soc. Japan
Vol. 36, No. 4, 1984

Enumerating embeddings of n-manifolds in Euclidean $(2n-1)$-space

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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(Received March 30, 1983)

Introduction.

Throughout this paper, "n-manifold" and "embedding" will mean closed connected differentiable manifold of dimension n and differentiable embedding, respectively. Let $[M\subset R^{m}]$ denote the set of isotopy classes of embeddings of a manifold M into Euclidean m-space.

It is known that for an n-manifold M,

(1) (Whitney [24]) the set $[M\subset R^{2n+2}]$ consists of only one element if $n\geqq 1$,

(2) (Wu [25]) the set $[M\subset R^{2n+1}]$ consists of only one element if $n\geqq 2$,

(3) (Haefliger [6], Bausum [1], Rigdon [13] etc.) if $n\geqq 4$, then, as a set,

$$[M\subset R^{2n}]=\begin{cases} H^{n-1}(M;\ Z) & \text{for } n\equiv 1\,(2),\ w_1(M)=0, \\ H^{n-1}(M;\ Z_2) & \text{for } n\equiv 1\,(2),\ w_1(M)\neq 0, \\ & \quad\text{or } n\equiv 0\,(2),\ w_1(M)=0, \\ Z\times\rho_2 H^{n-1}(M;\ Z) & \text{for } n\equiv 0\,(2),\ w_1(M)\neq 0. \end{cases}$$

The purpose of this paper is to inquire into the question of whether or not the set $[M\subset R^{2n-1}]$ for an n-manifold M, if it is not empty, can be described in terms of the cohomology of M, its characteristic classes and the cohomology operations. We shall study $[M\subset R^{2n-1}]$ along the lines of Haefliger [5], [6].

Let X^2 be the product $X\times X$ of X and let ΔX be the diagonal in X^2. The cyclic group of order 2, Z_2, acts on X^2 via the map $t: X^2\rightarrow X^2$ defined by $t(x, y)=(y, x)$, where ΔX is the fixed point set of this action. The quotient space

$$X^{*}=(X^{2}-\Delta X)/Z_{2}$$

is called the reduced symmetric product of X. Let P^{m} denote the real projective space of dimension m ($m\leqq\infty$) and let

$$\xi: X^{*}\longrightarrow P^{\infty}$$

denote the classifying map of the double covering $X^2-\Delta X\rightarrow X^{*}$. Then the first Stiefel-Whitney class of this double covering is given by