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On the asymptotic behavior of incompressible viscous fluid motions past bodies

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§ 1. Introduction.

Let Ω be a domain exterior to a finite number of bodies in E_3 with the smooth boundary $\partial\Omega$. The motion of the incompressible viscous fluid in Ω is described by the following system of the Navier-Stokes equations for the velocity $\boldsymbol{u}=(u_1(x, t), u_2(x, t), u_3(x, t))$ of the fluid and the pressure $\boldsymbol{p}=\boldsymbol{p}(x, t)$;

$$(1.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}\boldsymbol{u}-\nu\Delta\boldsymbol{u}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}+\operatorname{grad}\boldsymbol{p}=0 \\ \operatorname{div}\boldsymbol{u}=0 \end{cases} \qquad (x, t)\in Q_T,$$

where ν is a positive constant — "viscousity constant", $(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=u_i\partial\boldsymbol{u}/\partial x_i$, $0<T\leqq\infty$ and Q_T is the space time region $\Omega\times(0, T)$. Here and in what follows we use the conventional rule of tensor that repeated suffix means the summation with respect to the suffix.

We consider a flow $\boldsymbol{u}$ satisfying initial-boundary conditions;

$$(1.2)\qquad \boldsymbol{u}(x, 0)=\boldsymbol{a}(x), \qquad x\in\Omega,$$

$$(1.3)\qquad \boldsymbol{u}(x, t)=\boldsymbol{b}(x, t) \qquad x\in\partial\Omega,\ 0\leqq t<T,$$

$$(1.4)\qquad \boldsymbol{u}(x, t)\to\boldsymbol{u}_\infty \quad \text{as} \quad |x|\to\infty, \qquad 0\leqq t<T,$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are given smooth and bounded functions such that $\operatorname{div}\boldsymbol{a}=0$ and $\boldsymbol{a}(x)=\boldsymbol{b}(x, 0)$ for $x\in\partial\Omega$, and $\boldsymbol{u}_\infty$ is a prescribed constant vector. We are mainly concerned with the decay rate of $|\boldsymbol{u}(x, t)-\boldsymbol{u}_\infty|$ as $|x|\to\infty$; for the existence of solutions, see [7], [11], [14], [15] and especially [9], [10], [17] and [18].

In the case that $\boldsymbol{b}(x, t)$ is independent of t, R. Finn [4, 5, 6] showed that if a stationary solution $\boldsymbol{u}_s$ of (1.1), (1.3), and (1.4) has finite Dirichlet norm; $\|\nabla\boldsymbol{u}_s\|_{L^2(\Omega)}<\infty$, and satisfies

$$(1.5)\qquad \boldsymbol{u}_s(x)=\boldsymbol{u}_\infty+O(|x|^{-\alpha})$$

where α is a constant, $\alpha>1/2$, then