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Singular hyperbolic systems, V. Asymptotic expansions for Fuchsian hyperbolic partial differential equations

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In this paper, we study the asymptotic behavior of solutions of Fuchsian hyperbolic partial differential equations (in Tahara [**9-III**]), and determine complete asymptotic expansions of solutions in $C^{\infty}((0, T)\times \boldsymbol{R}^{n})$ as $t\rightarrow+0$. Our result corresponds to the well-known result in the theory of ordinary differential equations with regular singularities.

Let $(t, x)\in[0, T)\times \boldsymbol{R}^{n}$ $(T>0)$ and let

$$P(t, x, \partial_{t}, \partial_{x})=t^{m}\partial_{t}^{m}+P_{1}(t, x, \partial_{x})t^{m-1}\partial_{t}^{m-1}+\cdots+P_{m}(t, x, \partial_{x})$$

be a linear partial differential operator of order m $(\geqq 1)$ with C^{∞} coefficients on $[0, T)\times \boldsymbol{R}^{n}$. Assume that P satisfies the following conditions:

(i) *order* $P_{j}(t, x, \partial_{x})\leqq j$ $(1\leqq j\leqq m)$,

(ii) *order* $P_{j}(0, x, \partial_{x})\leqq 0$ $(1\leqq j\leqq m)$.

Then, P is said to be a *Fuchsian type operator* with respect to t. Further, if P satisfies some hyperbolicity conditions, P is said to be a *Fuchsian hyperbolic operator* with respect to t. By (ii), $P_{j}(0, x, \partial_{x})$ $(1\leqq j\leqq m)$ are functions in x. We set $P_{j}(0, x, \partial_{x})=a_{j}(x)$ $(1\leqq j\leqq m)$. Then, the indicial polynomial $\mathcal{C}(\lambda, x)$ associated with P is defined by

$$\mathcal{C}(\lambda, x)=\lambda(\lambda-1)\cdots(\lambda-m+1)+a_{1}(x)\lambda(\lambda-1)\cdots(\lambda-m+2)+\cdots+a_{m}(x)$$

and the characteristic exponents $\rho_{1}(x), \cdots, \rho_{m}(x)$ of P are defined by the roots of the indicial equation $\mathcal{C}(\lambda, x)=0$ in λ.

In [**9-III**], we have solved the Cauchy problem in $C^{\infty}([0, T)\times \boldsymbol{R}^{n})$ for Fuchsian hyperbolic operators P under various assumptions of hyperbolicity. But, here, we want to consider the equation

$$\text{(S)} \qquad P(t, x, \partial_{t}, \partial_{x})u(t, x)=0$$

in $C^{\infty}((0, T)\times \boldsymbol{R}^{n})$ (not in $C^{\infty}([0, T)\times \boldsymbol{R}^{n})$) under the same assumptions as in

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