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On some representations of continuous additive functionals locally of zero energy

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Introduction.

Many fruitful studies have been produced before 1980 to generalize the classical Ito formula for Ito processes and for C^2-functions to more general processes than Ito processes (H. Kunita - S. Watanabe [**9**], P. A. Meyer [**12**] and so on) or to more general functions than C^2-functions (for example Tanaka's formula [**10**], [**12**]). These generalizations can be characterized as specific realizations of semimartingale decomposition due to J. L. Doob and P. A. Meyer; Semimartingale=martingale+process of bounded variation.

At the end of 1970's, noting that the square integrable martingale of zero quadratic variation is identically zero, M. Fukushima has introduced a new point of view where the Ito formula can be conceived as a decomposition into the sum;

(0.1) Martingale+process of zero quadratic variation, or into the sum;

(0.1′) Martingale+continuous additive functional (CAF) of zero energy.

In this conception, he has established a unique decomposition ([**2**], [**3**], [**5**]);

$$u(X_t)-u(X_0)=M_t^{[u]}+N_t^{[u]},\ M_t^{[u]}\in\mathring{\mathcal{M}}_{\mathrm{loc}},\ N_t^{[u]}\in\mathcal{N}_{\mathrm{loc}} \tag{0.2}$$

for the symmetric Markov process X_t and for any function $u\in\mathcal{F}_{\mathrm{loc}}$ where $u\in\mathcal{F}_{\mathrm{loc}}$ means that u belongs locally to the Dirichlet space associated with X_t. In (0.2) $\mathring{\mathcal{M}}_{\mathrm{loc}}$ denotes the family of martingale additive functionals locally of finite energy and $\mathcal{N}_{\mathrm{loc}}$ is the family of CAF's locally of zero energy.

In this direction, M. Yor [**21**] and the second author of the present paper [**19**] produced several concrete realizations of the decomposition of the type (0.1′) and gave some applications to the local time of one dimensional Brownian path. Some related topics have been discussed in [**7**] and in [**20**].

Once the decomposition (0.2) has been established for $u\in\mathcal{F}_{\mathrm{loc}}$, it is quite natural to ask if the decompositions for $u\in\mathcal{F}_{\mathrm{loc}}$ exhaust all possible decompositions of the form (0.2): In other words the question is to ask if for any given $N_t\in\mathcal{N}_{\mathrm{loc}}$ there exists $u\in\mathcal{F}_{\mathrm{loc}}$ such that $u(X_t)-u(X_0)-N_t\in\mathring{\mathcal{M}}_{\mathrm{loc}}$ holds.

In § 2 of this paper, we will attempt to answer the question in the affirma-