

On an invariant of homology lens spaces

By Shinji FUKUHARA

(Received Dec. 4, 1982)

(Revised April 13, 1983)

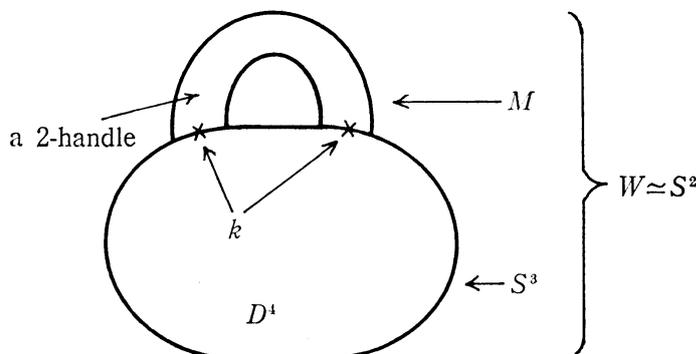
§1. Introduction.

In this paper we study problems concerning the following two questions:

QUESTION 1. Which 3-manifold (or lens space) can be obtained from S^3 by integer surgery on a knot?

QUESTION 2. Which 3-manifold (or lens space) bounds a compact 4-manifold that is a homology S^2 ?

These two questions are related each other. In fact, if a 3-manifold M is obtained from S^3 by integer surgery on some knot k , then M bounds a compact 4-manifold W which is homotopy equivalent to S^2 .



Concerning Question 2, Fintushel-Stern [F-S] observed that a lens space $L(p, q)$ bounds a homology S^2 only if q is a quadratic residue mod p . They and N. Maruyama showed that, in some special but non-trivial cases of p and q , $L(p, q)$ is obtained from S^3 by integer surgery on a knot. To the best of our knowledge, these are all known results for Questions 1 and 2.

We study the above questions for the 3-manifolds which have the same homology type as $L(p, 1)$. In §5 a certain new invariant is defined for some class of 3-manifolds and it is proved that vanishing of the invariant is necessary for such manifold to bound a homology S^2 . In §6 we calculate the invariant

This research was partially supported by Grant-in-Aid for Scientific Research (No. 58540044), Ministry of Education.