J. Math. Soc. Japan
Vol. 36, No. 2, 1984

Discrete reflection groups in a parabolic subgroup of Sp(2, $\boldsymbol{R}$) and symmetrizable hyperbolic generalized Cartan matrices of rank 3

By Masaaki YOSHIDA

(Received June 21, 1982)
(Revised March 28, 1983)

§ 0. Introduction.

Let Λ be a congruence subgroup of Siegel modular group Sp(2, $\boldsymbol{Z}$) acting on the Siegel upper half space $\mathfrak{S}_2$ of degree 2. Singularity and uniformizability of the Satake compactification of the factor space $\mathfrak{S}_2/\Lambda$ are studied by several authors (cf. for example, Igusa [2] and Christian [1]). In this paper, we shall study the uniformizability at the zero dimensional cusps.

Let $\mathcal{P}$ be the maximal parabolic subgroup of Sp(2, $\boldsymbol{R}$) corresponding to a zero dimensional boundary component. For a symmetrizable hyperbolic generalized Cartan matrix C of rank 3, we shall construct a discrete subgroup $\mathcal{P}(C)$ of $\mathcal{P}$, and show that the Satake compactification of the quotient space $\mathfrak{S}_2/\mathcal{P}(C)$ is non-singular by using Looijenga's theory ([4]).

The author wishes to express his deepest appreciation to Professor T. Ibukiyama and Professor T. Yamazaki who gave him unceasing encouragement and valuable comments.

§ 1. Generalized Cartan matrix and its Weyl group W.

We shall review some fundamental definition and facts about generalized Cartan matrices and their Weyl groups (see [4]).

DEFINITION. An $n\times n$ matrix $C=(C_{ij})$ is called a generalized Cartan matrix (G.C.M.) of rank n if (i) $C_{ij}\in\boldsymbol{Z}$, $C_{jj}=2$, (ii) $C_{ij}\leqq 0$ $(i\neq j)$ and (iii) $C_{ij}=0$ if and only if $C_{ji}=0$.

DEFINITION. The matrix C is said to be classical if C is the Cartan matrix of some semi-simple Lie algebra over $\boldsymbol{C}$.

DEFINITION. The matrix C is said to be euclidean if (i) it is indecomposable, (ii) $\det C=0$ and (iii) $C_k=(C_{ij})_{i,j\neq k}$ is classical for all k.