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Sullivan-Quillen mixed type model for fibrations

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§ 1. Introduction.

D. Sullivan [**10**] associated a differential graded algebra (DGA, for brevity) M_X over the rationals Q to a topological space X, while D. Quillen [**6**] associated to X a differential graded Lie algebra (DGL, for brevity) L_X over Q. These algebraic objects contain all the informations about the rational homotopy type of X at least when X has the homotopy type of a simply connected CW complex of finite homology type. The DGA M_X (and respectively, DGL L_X) is called a Sullivan (and respectively, Quillen) type (algebraic) model for the space X.

These assignments are generalized to the case of a fibration $F\to E\to B$, where we can associate a sequence of homomorphisms $M_B\to M_B\underset{\tau}{\otimes}M_F\to M_F$ of DGA's, or $L_F\to L_F\underset{\phi}{\oplus}L_B\to L_B$ of DGL's [**11**] with $M_B\underset{\tau}{\otimes}M_F$ (and respectively, $L_F\underset{\phi}{\oplus}L_B$) a Sullivan (and respectively, Quillen) type model for the total space E of the fibration.

Mixing these two types, A. Haefliger [**3**] constructed a DGL $M_B\underset{\tau}{\otimes}L_F$ over M_B which is free as an M_B-module and whose quotient by the ideal (M_B^+) is L_F, in order to prove the Bott conjecture concerning the Gelfand-Fuks cohomology. An advantage of his mixed type model lies in the following facts; the Koszul cochain complex construction over M_B of $M_B\underset{\tau}{\otimes}L_F$ gives rise to a Sullivan type model for the total space E while the same construction over the ground field R gives a Sullivan type model for the space of cross-sections of the fibration. A technical disadvantage of the mixed type model is that the base is negatively graded while the fiber is positively graded and that we have to consider a projective system when both of the base and fiber contain nontrivial elements of arbitrary high (low) degrees.

In this note we present basic definitions concerning the Sullivan-Quillen mixed type and the Sullivan type algebraic fibrations in section 2, and prove the equivalence of the homotopy categories of mixed type and Sullivan type fibrations in section 5, using the generalized Koszul complex functor C_A^* defined in section 3 and the generalized Quillen L functor L_*^A defined in section 4. This generalizes Silveira's result [**9**] on the mixed type model for fibrations with a given cross-