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Invariant spherical distributions and the Fourier inversion formula on $GL(n, \boldsymbol{C})/GL(n, \boldsymbol{R})$

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Introduction.

The purpose of this paper is to study the invariant spherical distributions on the reductive symmetric space $M \cong GL(n, \boldsymbol{C})/GL(n, \boldsymbol{R})$, and prove the Fourier inversion formula (the expansion of the δ-function by invariant spherical distributions on M) by using the same method as in the case of semisimple Lie groups, in particular by using the gap relations.

In more detail, the content of this paper is as follows. In the first place, we give the Weyl integral formula for general semi-simple symmetric spaces in § 1 and § 2. From § 3, we restrict ourselves to the space $GL(n, \boldsymbol{C})/GL(n, \boldsymbol{R})$. We define Harish-Chandra transforms on $\mathfrak{gl}(n, \boldsymbol{C})/\mathfrak{gl}(n, \boldsymbol{R})$ and on $M \cong GL(n, \boldsymbol{C})/GL(n, \boldsymbol{R})$ and prove the gap relations of these transforms in § 3 and § 4. Let A be a global Cartan subspace of M. Put $\mathfrak{g} = \mathfrak{gl}(n, \boldsymbol{C})$. Let $U(\mathfrak{g})$ be the universal enveloping algebra of $\mathfrak{g}$ and $\mathfrak{Z}$ be the center of $U(\mathfrak{g})$. The radial component of any element $D \in \mathfrak{Z}$ on A under the transformation of $GL(n, \boldsymbol{R})$ is given in Proposition 5.3. After establishing this proposition, we define invariant spherical distributions on M.

The Fourier inversion formula for semi-simple Lie groups G can be regarded as the expansion of δ-function on G by characters of irreducible unitary representations of G. These characters are invariant eigendistributions on G. Hence the inversion formula is the expansion of δ-function by invariant eigendistributions on G. We will discuss harmonic analysis on M from this point of view.

Let M' be the set of all $\mathfrak{q}$-regular elements in M (cf. § 1). The conditions that an analytic function on M' can be extended to an invariant spherical distribution on M are given in § 6. And in § 7 we construct the tempered invariant spherical distributions which contribute to the inversion formula for M. It is remarkable that these invariant spherical distributions are fairly different from the characters of representations of semi-simple Lie groups. In § 8, we give the

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