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Basic properties of Brownian motion and a capacity on the Wiener space

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§ 1. Introduction.

Among basic properties of the one-dimensional Brownian motion, we consider the property of quadratic variation, nowhere differentiability, Lévy's Hölder continuity and the law of the iterated logarithm. We shall prove that these properties hold not only almost everywhere (a. e.) with respect to the Wiener measure but also quasi everywhere (q. e.), namely, except on a polar set, with respect to the Ornstein-Uhlenbeck process on the Wiener space. We shall also consider the d-dimensional Brownian motion and establish q. e. statements of the unattainability of a one point set (when $d \geqq 5$), the transience (when $d \geqq 5$) and the absense of double points (when $d \geqq 7$).

Concerning the property of quadratic variation, D. Williams has obtained such refinement from a. e. to q. e. by a direct consideration of the Ornstein-Uhlenbeck process ([8]). In this paper, we instead make use of the estimates of a capacity related to the Ornstein-Uhlenbeck operator. A useful means in carrying out the computation of the estimates is a chain rule of the Dirichlet norm for composite functions. The rule has been stated already in the context of the Malliavin calculus ([**3**], [8], [**10**]) and in relation to the Dirichlet forms ([**1**], [**6**]).

To be precise, let us consider the d-dimensional Wiener space (W, P) ; $W = W_0^d$ is the space of all continuous functions $w : [0, \infty) \to \boldsymbol{R}^d$ satisfying $w_0 = \mathbf{0}$ and P is the Wiener measure on W. W is endowed with the topology of uniform convergence on every finite interval. The expectation with respect to P is denoted by E. The t-th coordinate of $w \in W$ is designated by w_t or $b_t(w)$ or $b(t, w)$. $\{b_t, t \geqq 0\}$ performs the d-dimensional standard Brownian motion under the law P. The inner product in $L^2 = L^2(W, P)$ is denoted by (,).

Let $L^2 = \sum_{n=0}^{\infty} \oplus Z_n$ be the Wiener-Ito decomposition, Z_n being the space of n-ple Wiener integrals. We consider a self-adjoint operator $\boldsymbol{A}$ on L^2 defined by

$$(1.1) \qquad \boldsymbol{A} = -\sum_{n=0}^{\infty} \frac{n}{2} P_n \qquad (P_n \text{ is the projection on } Z_n).$$

$\boldsymbol{A}$ is non-positive definite and consequently we may consider the associated closed