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A remark of decompositions of the regular representations of semi-direct product groups

Dedicated to Professor Hisaaki Yoshizawa on his 60th birthday

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Introduction.

The aim of the present paper is to show that the regular representations of some non-type I semi-direct product groups can be decomposed into direct integrals of irreducible representations in an uncountably infinite number of completely different ways. This is related with some cohomology groups.

The non-uniqueness of irreducible decompositions of a non-type I representation has been pointed out by several authors, for example, [**3**], [**4**], [**7**], [**8**], [**9**], [**10**], [**11**], [**12**], [**13**], [**18**], [**19**] and [**20**]. Concerning the regular representations λ of non-type I semi-direct product groups G, [**4**], [**12**] and [**13**] gave two kinds of entirely different irreducible decompositions of λ under some restrictions. In the present paper, we shall establish similar facts in a more general situation. We have studied in [**7**] and [**10**] that it is possible to give various kinds of irreducible decompositions of certain non-type I factor representations, related with some cohomology groups. In the present paper, we shall show that similar results may be obtained even for the regular representation λ of G and that there are an uncountably infinite number of completely different irreducible decompositions of λ in some cases.

Our main result is as follows. Let G be a semi-direct product $N\times_s K$ of N with K where N and K are assumed to be separable locally compact abelian groups. Then, the left regular representation λ of G is decomposed into irreducible components as

$$\lambda\cong\int_{\hat{N}}^{\oplus}\int_{\hat{H}_\chi}^{\oplus} U^{(\chi,\theta)}d\tau_\chi(\theta)d\mu(\chi) \qquad \text{(I)}$$

$$\cong\int_{Z}^{\oplus}\int_{\hat{K}}^{\oplus} V^{(a,\eta,\zeta)}d\nu(\eta)d\sigma(\zeta) \qquad \text{(II)}$$

where a is a cocycle of the double transformation group $(K;\hat{N}\times K;K)$. Further, we describe a maximal abelian von Neumann subalgebra A^a in $\lambda(G)'$ explicitly, which will give rise to the decomposition in (II). We state also the unitary in-