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Extension of modifications of ample divisors on fourfolds

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Introduction.

In this article we consider the following problem: Let A be an ample divisor on a connected four dimensional projective manifold X. Assume that the Kodaira dimension of X is non-negative. Suppose that A is the blow up of a projective manifold A' with center R_g where R_g is a smooth curve of genus $\geqq 1$ which is contained in A'.

Does there exist a four dimensional manifold X' such that A' lies on X' as a divisor and such that X is the blow up of X' with center R_g?

The answer to this question turned out to be positive. In fact following Sommese's idea, see [**13**], we construct a divisor D on X with the following properties:

1) $D \cap A = Y$, where Y is the exceptional divisor on A over R_g

2) the natural projection $Y \to R_g$ can be extended to a surjective holomorphic map $\tilde{p}: D \to R_g$

3) $\tilde{p}$ makes D a $\boldsymbol{P}^2$-bundle over R_g where $\dim A' - \dim R_g = 2$. Moreover, each fibre f' of Y over $x \in R_g$ is a hyperplane on $F = \tilde{p}^{-1}(x) \cong \boldsymbol{P}^2$.

4) $[D]_F = \mathcal{O}_{\boldsymbol{P}^2}(-1)$.

The above is enough to ensure the existence of X' such that A' is a divisor on X' and X is the blow up of X' with center R_g, see [8].

The above problem, in a more general setting, was already considered by Sommese in [**14**] and by Fujita in [**3**]. In fact they set up the problem for a projective manifold X of any dimension and without any assumption on the Kodaira dimension of X. Sommese in [**14**] showed that when $\operatorname{codim}_{A'} R > 2$ then there is an analytic set of codimension one in X that satisfies the condition for it to be blown down if the map $\tilde{p}: X \to X'$ existed. Fujita in [**3**] showed that the problem could be solved in the case $\operatorname{codim}_{A'} R > 2$ where R is a submanifold of A' along which we blow up.

We need the non-negativity of the Kodaira dimension for the theorem to be true. In fact given any projective threefold A there is a $\boldsymbol{P}^1$-bundle X over A