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On the structure of polarized manifolds with total deficiency one, III

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Introduction.

In this article we will study polarized manifolds (M, L) with $d(M, L)=\Delta(M, L)=1$, as a continuation of [**F4**]. But the arguments are completely independent of part II of it, and little knowledge of part I is required here. Moreover we consider here positive characteristic cases too, with the help of [**F5**].

In § 13, the first section of this part III, we study the structure of the rational mapping defined by $|L|$. It follows that $g=g(M, L)\geqq 1$. In § 14, assuming $\mathrm{char}(\mathfrak{K})\neq 2$ for the ground field $\mathfrak{K}$ from this time on throughout in this paper, we establish a precise structure theorem for (M, L) with $g=1$. When $g\geqq 2$, in general, we do not have so precise a result as in the case $g=1$. So we consider the case in which any curve $C=D_1\cap\cdots\cap D_{n-1}$ obtained by taking general members $D_1, \cdots, D_{n-1}$ of $|L|$ successively is a hyperelliptic curve. Such (M, L) will be said to be *sectionally hyperelliptic* (note that this is always the case when $g=2$). In § 15, they are classified into three types $(-)$, (∞) and $(+)$. Precise structures of them are described in § 16, § 17 and § 18 respectively. In particular, it turns out that $n=\dim M=2$ in case of type $(+)$, $n\leqq g+1$ in case of type (∞), and (M, L) is a weighted hypersurface of degree $4g+2$ in $\boldsymbol{P}(2g+1, 2, 1, \cdots, 1)$ in case of type $(-)$. In any case M is simply connected if $\mathfrak{K}=\boldsymbol{C}$. Moreover, all the (M, L) of the same type ($(-)$, (∞) or $(+)$) and with the same n and g form a single deformation family. It is easy to calculate the number of moduli of it.

Thus, when $\mathrm{char}(\mathfrak{K})\neq 2$, the classification theory of polarized manifolds (M, L) with $\Delta(M, L)=1$ is complete except the case $d(M, L)=1$, $g(M, L)\geqq 3$ and (M, L) is not sectionally hyperelliptic. In particular, all the Del Pezzo manifolds are completely classified.

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