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Studies on Hadamard matrices with "2-transitive" automorphism groups

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§ 1. Introduction.

An Hadamard matrix H of order n is a $\{-1, 1\}$-matrix of degree n such that $HH^t=H^tH=nI$, where t denotes the transposition. It is known that n equals one, two or a multiple of four. In this paper we assume that n is greater than eight. For the basic fact on Hadamard matrices see [**1**] or [**7**]. Let P be the set of $2n$ points $1, 2, \cdots, n, 1^*, 2^*, \cdots, n^*$. Then we define an n-subset α_i of P as follows: α_i contains j or j^* according as the (i, j)-entry of H equals $+1$ or -1 $(1\leqq i, j\leqq n)$. Let $\alpha_i^*=P-\alpha_i$. We call α_i and α_i^* blocks $(1\leqq i\leqq n)$. Let B be the set of $2n$ blocks $\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1^*, \alpha_2^*, \cdots, \alpha_n^*$. Then $M(H)=(P, B)$ is called the matrix design of H. By definition each point belongs to exactly n blocks. By the orthogonality of columns of H each point pair not of the shape $\{a, a^*\}$ belongs to exactly $n/2$ blocks, and each point trio not containing a point pair of the shape $\{a, a^*\}$ belongs to exactly $n/4$ blocks. $\{a, a^*\}$ does not belong to any block. Similarly by the orthogonality of rows of H each block pair not of the shape $\{\alpha, \alpha^*\}$ intersects in exactly $n/2$ points, and each block trio not containing a block pair of the shape $\{\alpha, \alpha^*\}$ intersects in exactly $n/4$ points.

We assume that $a^{**}=a$. Then $\alpha^{**}=\alpha$. Let $\mathfrak{G}$ be the group of all permutations σ on P such that σ leaves B as a whole. Then we call $\mathfrak{G}$ the automorphism group of $M(H)$. Obviously $\mathfrak{G}$ is isomorphic to the automorphism group of H. Since $\zeta=\prod_{a=1}^{n}(a, a^*)=\prod_{i=1}^{n}(\alpha_i, \alpha_i^*)$ belongs to the center of $\mathfrak{G}$, $\mathfrak{G}$ is imprimitive on P. For the basic facts on permutation groups see [**9**] or [**10**]. Now let $\bar{P}$ and $\bar{B}$ be the set of point pairs $\bar{a}=\{a, a^*\}$ and block pairs $\bar{\alpha}=\{\alpha, \alpha^*\}$, where $a\in P$ and $\alpha\in B$, respectively. Then $\mathfrak{G}$ may be considered as permutation groups on $\bar{P}$ and on $\bar{B}$. We notice that ζ is trivial on $\bar{P}$ and on $\bar{B}$, and that there is no apparent incidence relation between $\bar{P}$ and $\bar{B}$. In this paper we assume that $\mathfrak{G}$ on $\bar{P}$ is doubly transitive and that $\mathfrak{G}$ on $\bar{P}$ contains a regular normal subgroup $\mathfrak{N}$ on $\bar{P}$. Then $\mathfrak{N}$ on $\bar{P}$ is an elementary Abelian 2-group of order n, and so n

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