J. Math. Soc. Japan
Vol. 36, No. 1, 1984

Cancellation law for Riemannian direct product

By Kagumi UESU

(Received Sept. 11, 1980)
(Revised Nov. 29, 1982)

§0. Introduction.

L. S. Charlap showed that there are two compact differentiable manifolds M and N such that $M\times S^1$ is diffeomorphic to $N\times S^1$, while M and N are of different homotopy type (see [1]).

On the other hand, considering a Riemannian analogue of the above problem, we obtained the following result [3]:

Let M and N be connected complete Riemannian manifolds and S a connected compact locally symmetric Riemannian manifold. If $M\times S$ is isometric to $N\times S$, then M is isometric to N.

Later on, H. Takagi obtained the following result [2]:

Let M and N be connected complete Riemannian manifolds and let S be a connected complete Riemannian manifold which is simply connected or has the irreducible restricted homogeneous holonomy group. If $M\times S$ is isometric to $N\times S$, then M is isometric to N.

The purpose of this paper is to give a complete answer to the above problem in Riemannian case.

The main result is the following.

THEOREM. *If $M\times S$ is isometric to $N\times S$, then M is isometric to N, where M, N and S are connected complete Riemannian manifolds.*

In this paper, Riemannian manifolds are always supposed to be connected and complete, and $\cong$ means isometric.

We shall give a brief account of the idea of the proof. Let M, N and S be Riemannian manifolds such that $M\times S$ is isometric to $N\times S$. Then $M\cong X/\Gamma_1$, $N\cong X/\Gamma_2$ and $S\cong Y/\Gamma_3$ where X and Y are simply connected Riemannian manifolds and Γ_1, Γ_2 and Γ_3 are deck transformation groups of M, N and S, respectively. If we could find an isometry $\tilde{g}$ of $X\times Y$ satisfying Conditions 1 and 2 in Lemma 3, then our theorem would be proved. An isometry g of $X\times Y$ which is a natural lift of an isometry from $M\times S$ to $N\times S$ satisfies Condition 1 in Lemma 3. While if X and Y have the Euclidean parts in its de Rham decom-