J. Math. Soc. Japan
Vol. 36, No. 1, 1984

Residues of complex analytic foliation singularities

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(Received Nov. 15, 1982)

In [3], Baum and Bott defined the residues of complex analytic foliation singularities and proved a general residue formula using differential geometry based on the Bott vanishing theorem. Let M be a complex manifold. We define a foliation (of complete intersection type) on M to be a locally free subsheaf F of the cotangent sheaf Ω_M which satisfies the Frobenius integrability condition outside of the singular set (=the singular set of the coherent sheaf $\Omega_F=\Omega_M/F$). In this note, we express ((3.4) Theorem) a certain class of residues of F in terms of the Chern classes of F and the local Chern classes of the sheaf $\mathcal{E}xt^1_{\mathcal{O}}(\Omega_F, \mathcal{O})$, which appeared in the unfolding theory ([7]). As a corollary, the rationality of these residues is shown (cf. [3] p. 287 Rationality Conjecture). In a number of cases, the Riemann-Roch theorem for analytic embeddings (Atiyah-Hirzebruch [2]) can be used to compute the residues. The results of this paper were announced in [9].

This work was done while I was visiting University of Illinois at Chicago. I would like to thank especially J. Heitsch and P. Wagreich for their interest and helpful conversations. I am also grateful to H. Suzuki of Hokkaido University for reading the manuscript carefully and correcting some mistakes.

1. Residues.

We briefly review how the residues are defined in Baum-Bott [3]. Let M be an n-dimensional complex manifold. We denote by $\mathcal{O}_M$ (or simply by $\mathcal{O}$), Θ_M and Ω_M, respectively, the structure sheaf, the tangent sheaf and the cotangent sheaf of M. In [3] pp. 281-282, a foliation is defined to be a full integrable coherent subsheaf ξ of Θ_M. Let Q be the quotient sheaf Θ_M/ξ;

(1.1) $$0 \longrightarrow \xi \longrightarrow \Theta_M \longrightarrow Q \longrightarrow 0.$$

The singular set S of the foliation is defined by

(1.2) $$S=\{z\in M \mid Q_z \text{ is not a free } \mathcal{O}_z\text{-module}\},$$

This work was partially supported by the National Science Foundation.