

Non-separating incompressible tori in 3-manifolds

By Tsuyoshi KOBAYASHI

(Received Aug. 23, 1982)

(Revised Oct. 28, 1982)

1. Introduction.

In [1] Haken has shown that if a closed, connected 3-manifold M is not irreducible, then there exists such an essential 2-sphere in M that intersects a fixed Heegaard surface of M in a circle. Ochiai [4] has extended this result for a 2-sided projective plane in M . In this direction, we shall show that for a 2-sided, non-separating, incompressible torus and a genus two Heegaard splitting of M , the same result holds.

THEOREM 1. *Let M be a closed, connected (possibly, non-orientable) 3-manifold with a Heegaard splitting $(V_1, V_2; F)$ of genus two. Assume that M contains a 2-sided, non-separating, incompressible torus T . Then there exists a 2-sided, non-separating, incompressible torus T' which intersects F in a circle.*

As an application, we shall show that any orientable, closed 3-manifold which has a Heegaard splitting of genus two and contains a non-separating, incompressible torus is obtained by pasting boundary components of two bridge link space by a certain type of homeomorphism and performing a Dehn surgery along the two meridian loops of this link (Theorem 2).

As a consequence of Theorem 2, we have

COROLLARY. *If an orientable, closed 3-manifold M with a Heegaard splitting of genus two contains a non-separating, incompressible torus, then M is a 2-fold branched covering space of $S^2 \times S^1$ branched along a 1-manifold.*

I would like to express my gratitude to Prof. M. Ochiai for helpful conversations.

2. Preliminaries.

Throughout this paper, we will work in the piecewise linear category. A Heegaard splitting of a closed, connected 3-manifold M is a pair $(V_1, V_2; F)$, where V_i ($i=1, 2$) is a three dimensional orientable or nonorientable handlebody such that $M=V_1 \cup V_2$ and $V_1 \cap V_2 = \partial V_1 = \partial V_2 = F$. Then F is called a Heegaard surface of M . The first Betti number of V_1 is called the genus of the splitting.