J. Math. Soc. Japan
Vol. 36, No. 1, 1984

High energy resolvent estimates, II, higher order elliptic operators

By Minoru MURATA

(Received May 6, 1982)

§ 1. Introduction.

This is a continuation of [1] and is concerned with elliptic partial differential operators on $\boldsymbol{R}^n$ whose coefficients are nearly constants at infinity. For such an operator $A(X, D_x)$ with real principal symbol $a(x, \xi)$ we shall show in this paper that if every classical orbit under the Hamiltonian $a(x, \xi)$ is not trapped, then the resolvent of $A(X, D_x)$ admits, as operators from a weighted L_2-space to its dual space, boundary values on the upper or lower bank of the reals which are bounded and uniformly Hölder continuous at infinity; and that the nontrapping condition for orbits is necessary to the uniform estimate for the resolvent.

In connection with time-decay for solutions of Schrödinger-type equations uniform estimates at high energy for resolvents of elliptic operators have been investigated (see [2], [4], [5], and references there). But such estimates were given only for operators whose leading and the next coefficients are constant. The purpose of this paper is to give high energy resolvent estimates for elliptic differential operators with variable leading coefficients. The results here will be used in [3] to study the asymptotic behavior as $t\rightarrow\infty$ of solutions for Schrödinger-type equations.

Now we prepare some notations in order to state our main results. We write $D_{x_j}=-i\partial/\partial x_j$, $D_x=(D_{x_1}, \cdots, D_{x_n})$, and $\langle x\rangle=(1+|x|^2)^{1/2}$. For a real number σ and s, $H^{\sigma,s}$ denotes the weighted Sobolev space with the norm

$$\|f\|_{\sigma,s}\equiv\|f\|_{H^{\sigma,s}}=\|\langle x\rangle^s\langle D_x\rangle^\sigma f\|_{L_2(R^n)}\,. \tag{1.1}$$

We write $H^\sigma=H^{\sigma,0}$ and $L_2^s=H^{0,s}$. $B(\sigma, s\,;\, \tau, t)$ stands for the Banach space of all bounded linear operators from $H^{\sigma,s}$ to $H^{\tau,t}$. We write $B(s, t)=B(0, s\,;\, 0, t)$. For a positive number h and a function f on $\boldsymbol{C}$ to a Banach space, we put

$$\Delta_h^1 f(x)=f(x+h)-f(x)\,, \qquad \Delta_h^2 f(x)=f(x+2h)-2f(x+h)+f(x)\,.$$

For a real number r, $[r]$ denotes the largest number which is not larger than r.

We consider the elliptic partial differential operator