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Value distribution of the Gauss maps of complete minimal surfaces in $\boldsymbol{R}^m$

Dedicated to Professor M. Ozawa on the occasion of his 60th birthday

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§1. Introduction.

Concerning the value distribution of the Gauss maps of complete minimal surfaces in $\boldsymbol{R}^m$, there have been several results obtained by R. Osserman, S.S. Chern, F. Xavier and others ([10], [2], [7], [13]). Recently, the author proved that the Gauss map of a complete minimal surface in $\boldsymbol{R}^m$ is necessarily degenerate if it omits more than m^2 hyperplanes in $P^{m-1}(\boldsymbol{C})$ located in general position ([4]). The purpose of this paper is to give several improvements of these results.

Let f be a holomorphic map of an open Riemann surface M into $P^n(\boldsymbol{C})$ and H a hyperplane in $P^n(\boldsymbol{C})$ with $f(M)\not\subset H$. For an arbitrarily fixed positive integer μ_0 we define the non-integrated defect of H for f by

$$\delta_{\mu_0}^f(H):=1-\inf\{\eta\geqq 0 : \eta \text{ satisfying condition } (*)\}.$$

Here, condition $(*)$ means that there exists a non-negative smooth function v on M such that $\log v$ is subharmonic, $\log v\leqq\eta\log\|f\|$ and, in a neighborhood of each point $p\in f^{-1}(H)$,

$$\log v(\zeta)-\min(\nu^f(H)(p),\ \mu_0)\log|\zeta-\zeta(p)|$$

is subharmonic, where $\|f\|:=(|f_1|^2+\cdots+|f_{n+1}|^2)^{1/2}$ for a reduced representation $f=(f_1:\cdots:f_{n+1})$, ζ is a holomorphic local coordinate around p and $\nu^f(H)(p)$ denotes the intersection multiplicity of $f(M)$ and H at $f(p)$. We note that

$$\delta_{\mu_0}^f(H)=1 \tag{1.1}$$

if $f(M)\cap H=\emptyset$, or more generally, if there is a bounded holomorphic function g on M such that g has zeros of order $\nu^f(H)(p)$ at each point $p\in f^{-1}(H)$. Moreover, we can show that

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