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On the sample continuity of $\mathscr{S}'$-processes

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1. Introduction and results.

Let E be a nuclear Fréchet space and E' the topological dual space (of which the Schwartz space $\mathscr{S}'$ of tempered distributions is a typical one). We denote by $\langle x, \xi\rangle$, $x\in E'$, $\xi\in E$ the canonical bilinear form on $E'\times E$. Let $\boldsymbol{X}=\{X_t; t\in[0, \infty)\}$ be a stochastic process defined on a complete probability space $(\Omega, \mathfrak{F}, P)$ with values in E'. In the previous paper [**4**] the author showed that $X_1=\{X_t; t\in[0, 1]\}$ has a strongly continuous version if for each $\xi\in E$, the process $\langle X_t, \xi\rangle$ has a continuous version and satisfies the moment condition

$$\int_{\Omega}\sup_{t\in Q}|\langle X_t, \xi\rangle|^{\rho}dP<+\infty, \tag{1.1}$$

where $\rho>0$ and Q is a countable dense subset of $[0, 1]$.

In this paper, we will prove the similar results without assuming the moment condition. The results are stated as follows:

THEOREM 1. *Let E be a nuclear Fréchet space and $\boldsymbol{X}$ an E'-valued stochastic process such that for each ξ in E the real stochastic process $X_{\xi}=\{\langle X_t, \xi\rangle; t\in[0, \infty)\}$ has a continuous version. Then $\boldsymbol{X}$ has a strongly continuous version.*

THEOREM 2. *Let E be a nuclear Fréchet space and $\boldsymbol{X}$ an E'-valued stochastic process such that for each ξ in E the real stochastic process X_{ξ} has a version which is right continuous and has left-hand limits. Then $\boldsymbol{X}$ has a version which is right continuous and has left-hand limits in the strong topology of E'.*

The proof of Theorem 1 will be given in Section 2. The proof of Theorem 2 is quite similar to that of Theorem 1, so that we will omit it. As applications of Theorem 1, we will give a characterization of the existence of a continuous version with respect to a certain norm and a generalized Kolmogorov's criterion for continuity in Section 3.

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