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Sylow 2-intersections, 2-fusion, and 2-factorizations in finite groups of characteristic 2 type

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Introduction.

There is a close relationship between Sylow intersections, fusion, and factorizations in finite groups. This is probably best illustrated by the following examples. Let p be a prime and G be a group of order divisible by p. Define $\mathcal{H}_0$ to be the set of all nonidentity p-subgroups H of G such that $N_G(H)/H$ is p-isolated in the sense of Goldschmidt [**10**]. Let $\mathcal{N}_0$ be the set of the normalizers of the elements of $\mathcal{H}_0$. Then the following holds.

(1) *$\mathcal{N}_0$ controls Sylow p-intersections in G.*

(2) *$\mathcal{N}_0$ controls p-fusion in G.*

(3) *If G is not p-isolated and $S \in \mathrm{Syl}_p(G)$, then*

$$G = \langle N \in \mathcal{N}_0 ; S \cap N \in \mathrm{Syl}_p(N) \rangle N_G(S) .$$

In the above, (1) is essentially a lemma in [**11**, (2.3)], and the reader is referred to Kondo [**16**, Lemma 2] for a generalization of (1) and the precise meaning of 'control' in (1) (the definition of the control in the most general form will be given in the first section of the present paper). The proposition (2) is a theorem of Goldschmidt [**10**, Theorem 3.4] improving Alperin's fusion theorem [**1**]. The proposition (3) is considered to be a sort of p-factorization theorem, and is an easy consequence of (1). It has already been pointed out that (2) can easily be derived from (1) also [**12**, Proposition 2.4], [**16**, Theorem 1].

Still more interesting than (1), (2), and (3) are the following theorems of Aschbacher [**3**] and P. McBride.

(4) *If G is a group of characteristic 2 type, $S \in \mathrm{Syl}_2(G)$, and G is not generated by the normalizers of nontrivial characteristic subgroups of S, then either G is 2-isolated or some maximal 2-local subgroup of G has a block in $\mathfrak{X}$.*

(5) *If G is a group of characteristic 2 type in which each simple section of each 2-local subgroup is of known type and if 2-fusion in G is not controlled by the normalizers of nontrivial characteristic subgroups of a Sylow 2-subgroup of G, then some maximal 2-local subgroup of G has a block in $\mathfrak{X}$.*

In the above, (5) was announced at the A.M.S. Summer Institute held at the