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On simple groups which are homomorphic images of multiplicative subgroups of simple algebras of degree 2

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Let $M_2(D)$ be the full matrix algebra of degree 2 over a division algebra D of characteristic 0. In [**11**] we proved that if G is a finite multiplicative subgroup of $M_2(D)$ with abelian Sylow 2-subgroups, then G is a solvable group. More generally, in this paper we will determine the non-abelian simple groups S which are homomorphic images of multiplicative subgroups G of $M_2(D)$. In [**10**] we remarked that abelian subgroups of the Sylow 2-subgroups of G are generated by at most 2 elements. In particular, the Sylow 2-subgroups possess no abelian normal subgroups of rank 3, which implies that these 2-groups are generated by at most 4 elements (see MacWilliams [**14**]). All simple groups whose Sylow 2-subgroups are generated by at most 4 elements have been determined in Gorenstein-Harada [**7**]. Using their theorem, we will determine the simple groups S.

Our main result is as follows.

THEOREM. *Let S be a simple group. If there exists a division algebra D of characteristic* 0, *a finite multiplicative subgroup G of $M_2(D)$ and a normal subgroup N of G satisfying $G/N \cong S$, then S is isomorphic to $PSL(2, 5)$ or $PSL(2, 9)$ and $N \neq 1$.*

In the theorem $N \neq 1$ means the following:

COROLLARY. *Let G be a finite group and let K be a field of characteristic* 0. *If one of the simple components of the group ring KG is the full matrix algebra of degree* 2 *over a division algebra, then G is not simple.*

The corollary can not be generalized to the full matrix algebra of degree $\geqq 3$. In fact,

$$\boldsymbol{Q}[PSL(2, 5)] \cong \boldsymbol{Q} \oplus M_3(\boldsymbol{Q}(\sqrt{5})) \oplus M_4(\boldsymbol{Q}) \oplus M_5(\boldsymbol{Q})$$

and

$$\boldsymbol{Q}[A_n] \cong \boldsymbol{Q} \oplus M_{n-1}(\boldsymbol{Q}) \oplus \cdots, \quad n \geqq 5.$$

1. Preliminaries.

All division algebras considered in this paper are of characteristic 0. As usual $\boldsymbol{Q}$ and $\boldsymbol{C}$ denote respectively the rational number field and the complex