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# Some nonlinear degenerate diffusion equations related to population dynamics

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## 1. Introduction.

For the study of the spatial distribution of organisms, there are a large number of spatially spreading population models in which biological interactions and diffusion are taken into account. Among them, several models include nonlinear diffusion processes called "density-dependent dispersal". From ecological aspects, the works by Gurney and Nisbet [8], Gurtin and MacCamy [**9**] are relevent here.

In the category of such models, we propose a population model which provides a nonlocal interaction

$$u_t=(D(u)u_x)_x+\left[\left(\int_{-\infty}^{\infty}K(x-\xi)u(\xi, t)d\xi\right)u\right]_x, \tag{1.1}$$

where $u(x, t)$ denotes the population density at position $x\in \boldsymbol{R}^1$ and at time $t$, $D(u)$ is the diffusion rate satisfying $D(0)=0$ and $D'(u)>0$, and $K(x)$ is an odd function such that $K(x)>0$ for $x>0$. For one example, we have

$$K(x)=\begin{cases} ke^{-sx}, & x>0, \\ -ke^{-sx}, & x<0, \end{cases}$$

for a non-negative constant $s$. The second term of (1.1) ecologically implies a kind of aggregative mechanism of the individuals, which is motivated by the notion of "the selfish avoidance of a predator can lead to aggregation" (see, Hamilton [**10**]). If we restrict $D$ and $K$ to the specific forms $D(u)=mu^{m-1}$ and

$$K(x)=\begin{cases} k\,, & x>0, \\ -k\,, & x<0, \end{cases}$$

where $m>1$ and $k>0$ are constants, then (1.1) is rewritten as

$$u_t=(u^m)_{xx}+k\left[\left(\int_{-\infty}^{x}u(\xi, t)d\xi-\int_{x}^{\infty}u(\xi, t)d\xi\right)u\right]_x.$$

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