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Scattering of solutions of nonlinear Klein-Gordon equations in three space dimensions

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1. Introduction.

Let $h(u)$ be a C^1 function on $\boldsymbol{R}$ such that $h(0)=0$,

(H1) $$|h'(u)| \leqq c|u|^{p-1}, \qquad \forall u \in \boldsymbol{R},$$

and

(H2) $$H(u)=\int_0^u h(s)ds \geqq 0 \qquad \forall u \in \boldsymbol{R}$$

where $p>1$.

Consider the nonlinear Klein-Gordon equation (the "perturbed" equation)

(NLKG) $$\frac{\partial^2}{\partial t^2}u-\Delta u+m^2u+h(u)=0 \qquad (x \in \boldsymbol{R}^3,\ t \in \boldsymbol{R})$$

together with the "free" equation

(KG) $$\frac{\partial^2}{\partial t^2}v-\Delta v+m^2v=0, \qquad (x \in \boldsymbol{R}^3,\ t \in \boldsymbol{R})$$

where m is a positive constant.

In [**10**]-[**13**] W. Strauss developed the theory of nonlinear scattering at low energy, in which one looks for conditions under which solutions u of (NLKG) are related to free solutions $u_{\pm}$ of (KG) by the asymptotic condition $\|u(t)-u_{\pm}(t)\|_e \to 0$ as $t \to \pm\infty$, where $\|\cdot\|_e$ denotes the energy norm:

$$\|w(t)\|_e^2=\int\left[\left(\frac{\partial w}{\partial t}\right)^2+|\nabla w|^2+m^2w^2\right]dx \quad \left(=\left\|\left(w(t),\ \frac{\partial w}{\partial t}(t)\right)\right\|_e\right).$$

It has been shown that under the least regularity assumption on solutions, that is, under the assumption that solutions are of finite energy, the theory of nonlinear scattering at low energy holds in case of $1+\frac{4}{3} \leqq p \leqq 3$. If we require more regularity of solutions, the theory holds for $2<p<5$. In [3] Glassey showed that the theory does not hold if $1<p<1+\frac{2}{3}$.