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On spherical space forms which are isospectral but not isometric

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Introduction.

A compact connected Riemannian manifold of constant curvature 1 is said to be a *spherical space form*. If the fundamental group of a spherical space form is cyclic then the spherical space form is called a *lens space*.

Let M and N be spherical space forms. In papers [2], [3], [4] and [5], we studied the spectrum of Laplacian acting on smooth functions of a spherical space form and considered the following problem.

Whether or not M is isometric to N when M is isospectral to N?

In [5], we saw that there are many pairs of lens spaces which are isospectral but not isometric.

In this paper, we consider the above problem in cases which the fundamental groups of M and N are *noncyclic*. First we prove

THEOREM 1. *Let S^{2d-1}/G and S^{2d-1}/G' be spherical space forms with noncyclic fundamental groups of type* 1. *Suppose G and G' are irreducible and that G is isomorphic to G'. Then S^{2d-1}/G is isospectral to S^{2d-1}/G'.* (For the definitions of "type 1" and "irreducible" in Theorem 1, see Sections 2 and 3 respectively).

From this Theorem we can show that there are many pairs of spherical space forms with *noncyclic fundamental groups* which are isospectral but not isometric (for more precise statement, see Theorem 3). And moreover we see that there are spherical space forms which are isospectral but not isometric in every odd dimension not less than 5 (see Theorem 4).

Two lens spaces which are isospectral but not isometric are also not homeomorphic to each other (see [5]). Moreover we obtained in [5] examples of pairs of lens spaces which are isospectral but not even homotopically equivalent. But unfortunately the author don't know whether there are any topological differences between these isospectral non-isometric spherical space forms with noncyclic fundamental groups.

REMARKS. 1. As we have shown in [3], every 3-dimensional spherical