J. Math. Soc. Japan
Vol. 35, No. 2, 1983

Remarks on Sobolev inequalities and stability of minimal submanifolds

Dedicated to Professor Shigeo Sasaki on his 70th birthday

By Shukichi TANNO

(Received Feb. 27, 1982)

Sobolev inequalities for Riemannian submanifolds have applications to isoperimetric inequalities, estimates of the first eigenvalue, and the stability of minimal submanifolds. Here we give an improvement of constant of the Sobolev inequality by Hoffman-Spruck-Otsuki and apply it to the stability of minimal submanifolds.

Since the estimates of stability using the Sobolev inequality require the condition that the volume of a domain D is very small, our improvement is now effective. Especially, for the case where $\dim D=2$, to get stability estimate one needs additional estimates, i.e., the estimation of the first eigenvalue. So Hoffman's stability theorem (Theorem 5, (ii), [1]) contains much loss. In this article we carry the volume estimation and we get a nice improvement as Theorem D. As a corollary we obtain

COROLLARY. *Let M be a minimal surface of a unit sphere S^n and D be a compact domain of M. If*

$$\int_D (4-2K)^2 dM < 1/2c_3(2, \alpha_2)^2,$$

then D is stable in S^n, where K denotes the Gauss curvature of M and

$$\alpha_2=(9-\sqrt{57})/2,$$

$$\gamma^{-1}=\{\mathrm{Vol}(D)/(1-\alpha_2)\pi\}^{1/2},$$

$$c_3(2, \alpha_2)=\gamma\cdot\sin^{-1}(1/\gamma)\cdot 2(3-\alpha_2)/\alpha_2(1-\alpha_2)^{1/2}\pi^{1/2}.$$

§1. Sobolev inequality for submanifolds.

First we state a Sobolev inequality for submanifolds obtained by D. Hoffman-J. Spruck [2] and T. Otsuki [4]. Let $M\to\bar{M}$ be an isometric immersion of a

This research was partially supported by Grant-in-Aid for Scientific Research (No. 56540012), Ministry of Education.