J. Math. Soc. Japan
Vol. 35, No. 2, 1983

On the holomorphic equivalence of bounded domains in complete Kähler manifolds of nonpositive curvature

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(Received Jan. 4, 1982)

1. Introduction.

Suppose D_1 and D_2 are two bounded domains in the complex n-space $\boldsymbol{C}^n$, $n \geqq 2$, with C^∞ boundaries ∂D_1 and ∂D_2, respectively. One of the fundamental problems in several complex variables is to determine geometric conditions which imply that D_1 and D_2 are biholomorphically equivalent. It has been known from Bochner-Hartogs' theorem (Bochner [1]) that if ∂D_1 and ∂D_2 are connected and CR-diffeomorphic, then D_1 and D_2 are biholomorphically equivalent; and moreover by the second author [7] that the same is the case even for those domains in Stein manifolds.

In this paper we are concerned with the problem for domains in complete Kähler manifolds of nonpositive curvature. Our result is stated as follows.

THEOREM. *Let M and N be complete Kähler manifolds of complex dimension $n \geqq 2$. Let $D_1 \subset M$ and $D_2 \subset N$ be relatively compact subdomains in M and N with C^∞ boundaries ∂D_1 and ∂D_2, respectively. Suppose that* (i) *N has adequate negative curvature in the sense of Siu* [8], (ii) *the boundary ∂D_1 is pseudoconvex, and* (iii) *there exists a* CR*-diffeomorphism $f : \partial D_1 \to \partial D_2$ which extends to a homotopy equivalence of D_1 to D_2. Then D_1 and D_2 are biholomorphically equivalent. In fact, f extends to a biholomorphic diffeomorphism of D_1 to D_2.*

The adequate negativity of curvature, assumed in the hypothesis (i), is in fact stronger than requiring nonpositive sectional curvature. It is, however, known by Siu [8] that the classical bounded symmetric domains with their invariant metrics and their quotient Kähler manifolds have adequate negative curvature. It should be remarked that the curvature hypothesis (i) is assumed only on the target manifold N.

Some results related to ours can be seen in Wood [9]. We wish to thank him for making his manuscript available during the preparation of this paper.

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* Partially supported by Grant-in-Aid for Scientific Research No. 574018, Ministry of Education.