

Behavior of geodesics in foliated manifolds with bundle-like metrics

By Shinsuke YOROZU

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1. Introduction.

Foliated manifolds are studied by C. Ehresmann, A. Haefliger, G. Reeb and many people. Many of works are topological (non-riemannian) cases. The early study of riemannian case was done by B.L. Reinhart [24], that is, he defined foliated manifolds with "bundle-like" metrics with respect to the foliations and proved so-called Reeb stability theorem for this case. The foliated manifolds with bundle-like metrics are studied by R. Hermann [4], A.M. Naveira [19], J.S. Pasternack [22, 23], B.L. Reinhart [24, 25], R. Sacksteder [26], I. Vaisman [28, 29] and others.

The typical examples of foliated manifolds with bundle-like metrics are the followings; (i) each fiber space under a suitable choice of metric, (ii) the foliation of a riemannian manifold by orbits of a group of isometries having all its orbits of the same dimension.

In this paper we discuss the behavior of geodesics in foliated manifolds with bundle-like metrics. As a well-known and fundamental result in this direction, we may state:

THEOREM (B.L. Reinhart [24]). *A geodesic of a bundle-like metric is orthogonal to the leaf at one point if and only if it is orthogonal to the leaf at every point.*

We discuss geodesics making constant angles with leaves, and these are generalizations of [14]. We discuss focal points of leaves along transversal geodesics, and, in the case of codimension 1, we have non-existence of focal points of leaves along transversal geodesics. The relations between the Levi-Civita connection and the second connection defined by I. Vaisman [28] are discussed.

The topological obstructions for the existence of the foliation with a bundle-like metric were studied by H. Kitahara and S. Yorozu [12], J.S. Pasternack [22] and R. Sacksteder [26]. The existence of the complete bundle-like metric

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