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On the class of polar sets for a certain class of Lévy processes on the line

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Let X be a Lévy process (a process with stationary independent increments) on the line having the exponent Ψ and λ-capacity function C^{λ}. We assume that X satisfies the following conditions:

(A_1) *the λ-resolvent is absolutely continuous with respect to Lebesgue measure,*

(A_2) *every point is polar,*

(D_α) *for a fixed $\lambda>0$ there exist α $(1>\alpha>0)$, and a continuous function F on $(0, \infty)$ such that*

$$F(z)\asymp\mathcal{R}e([\lambda+\Psi(z)]^{-1}), \qquad z\rightarrow\infty,$$

and $z^{\alpha}F(z)$ is decreasing on $(0, \infty)$, and

(I) *for a fixed $\lambda>0$ there exists a constant $M>0$ such that*

$$\mathcal{R}e([\lambda+\Psi(2z)]^{-1})/\mathcal{R}e([\lambda+\Psi(z)]^{-1})\geqq M$$

for every $z>0$.

Throughout this article we use the notation $f(z)\prec g(z),\ z\rightarrow a$, if $\limsup_{z\rightarrow a} f(z)/g(z)<\infty$ and $f(z)\asymp g(z),\ z\rightarrow a$, if $f(z)\prec g(z),\ z\rightarrow a$ and $g(z)\prec f(z),\ z\rightarrow a$. We write $f(z)\ll g(z),\ z\rightarrow a$, if $\lim_{z\rightarrow a} f(z)/g(z)=0$.

Then we have

THEOREM 1. *Suppose that X satisfies* (A_1), (A_2), (D_α) *and* (I). *Put*

$$\phi(x)=\int_0^{1/x}\mathcal{R}e([\lambda+\Psi(z)]^{-1})\,dz, \qquad x>0.$$

Then $C^{\lambda}(K)=0$ if and only if $C^{\phi}(K)=0$, where $C^{\phi}(K)$ denotes the Frostman's ϕ-capacity of K.

For general class of Lévy processes, if, for $\lambda>0$ and $M_1>0$

$$\mathcal{R}e([\lambda+\Psi_1(z)]^{-1})\leqq M_1\mathcal{R}e([\lambda+\Psi_2(z)]^{-1}) \qquad \text{for all } z, \tag{0.1}$$

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