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On the local Hecke series of some classical groups over $\mathfrak{p}$-adic fields

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Introduction.

The purpose of this paper is to prove the rationality for the local Hecke series of some classical groups over $\mathfrak{p}$-adic fields, and to calculate the degrees of its numerator and the denominator.

Let k be a $\mathfrak{p}$-adic field. Let K be either k itself, a quadratic extension of k, or the (unique) central division quaternion algebra over k. We denote by $x \mapsto \bar{x}$ $(x \in K)$ the canonical involution. Let ε be an element of the center of K such that $\varepsilon\bar{\varepsilon}=1$, V be an n-dimensional (right) vector space over K with a non-degenerate ε-hermitian form $\Phi(\ ,\ )$, and L be a maximal lattice in V (cf. §1-1). Let G be the connected component (in the sense of an algebraic group over k) of

$$\tilde{G}=\{g \in GL(V)\,;\ \Phi(gx, gy)=\mu(g)\,\Phi(x, y) \text{ for all } x, y \in V,\ \mu(g) \in k^{\times}\}.$$

Let U be the subgroup of G consisting of all elements of G which leave L invariant. It is known that U is a maximal compact subgroup of G. For $m \geqq 0$, set

$$X(m)=\{g \in G\,;\ gL \subset L,\ \mathrm{ord}_{\mathfrak{p}}\mu(g)=fm\},$$

where $\mathrm{ord}_{\mathfrak{p}}(x)$ is the $\mathfrak{p}$-order of x for $x \in k$, and the positive integer f is determined by the condition $\mathrm{ord}_{\mathfrak{p}}\mu(G)=f\boldsymbol{Z}$. Let $T(m)$ be the characteristic function of $X(m)$ in G, considered as an element of the Hecke algebra of the group G with respect to U (see §1-2). Then the (local) Hecke series of the group G with respect to U is by definition

$$Z_{(G,U)}(T)=\sum_{m=0}^{\infty} T(m)\,T^{m},$$

where T is an indeterminate.

Our main result is that the Hecke series $Z_{(G,U)}(T)$ is a rational function in T, and the degree of the numerator is $2^{\nu}-1$ or $2^{\nu}-2$, while that of the denominator is 2^{ν}, where ν is the Witt index of (V, Φ).

When Φ is an alternating form, the Hecke series has been studied in