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Group extensions and Plancherel formulas

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§1. Introduction.

The purpose of this paper is to describe the Plancherel formula for some locally compact groups and to investigate the associated objects through the intermediary of a suitable normal subgroup and something related with it.

A. Kleppner and R. L. Lipsman ([**12**], [**13**]) discussed this problem under the assumptions that a normal subgroup N of G is "essentially" of type I (cf. Definition 5-1 for detail), the action of G on $\hat{N}$ is smooth, and G is isotropically of type I almost everywhere. We can regard their results as a "little group analysis" in the Plancherel formula context.

In this paper, when N is "essentially" of type I, instead of Kleppner and Lipsman's smoothness (type I'ness) condition, we assume that the action of G on $\hat{N}$ is locally essentially free (Definition 5-4). Whereas it is out of extent of the Mackey theory, we can do the "little group analysis" about the Plancherel objects. We will be mainly interested in the non type I groups as the subjects of this extended analysis.

The (central) decomposition of the Haar weight on $C^*(G)$ into Δ_G-semicharacters is regarded as the Plancherel formula. A measure (class) which gives the central decomposition of the left regular representation of G and so gives the Plancherel formula of G is called Plancherel measure (class). Since the Haar measure of G is Δ_G-relatively invariant with respect to inner automorphism, the above "Plancherel formula" can be regarded as the "global duality" of G.

In order to establish the theory of decompositions of Haar weight, we must make free use of the inductions and the direct integral decompositions of semitraces. We discuss these matters in §2. The author has received a recent preprint of N. V. Pedersen (On the left regular representations of locally compact groups), after having finished the preparation of this paper, which contains discussions of similar problems but the conclusions are slightly different. In contrast to Pedersen, we used and refined the decomposition of left Hilbert algebra established by C. E. Sutherland [**21**]. Moreover, in this section we discussed the case of projective semitraces in order to treat the problems in the group extension situation more closely in future.