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On the degree of symmetry of a certain manifold

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Introduction.

In their paper [**10**], R. Schoen and S. T. Yau have studied compact Lie group actions on the manifold which admits a map of degree one into a Riemannian manifold with non-positive sectional curvature. One of our purpose of this paper is to prove the topological part of results of Theorem 7 in [**10**] without differential geometrical methods. Since a Riemannian manifold with non-positive sectional curvature is *aspherical*, i.e. a manifold whose universal covering is contractible, we restrict ourselves to manifolds which admit a map of degree one into an aspherical manifold. In this note, we shall first prove a result which is analogous to [**5**] and then apply it to the study of a compact connected Lie group action on the manifold which admits a map of degree one into an aspherical manifold. We shall also consider the degree of symmetry of a connected sum $M\sharp N$, where M is a closed manifold and N is an aspherical manifold.

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In this note, we shall only consider continuous action and the term "*manifold*" will mean compact connected topological manifold without boundary. Note that manifolds have the homotopy type of a finite CW complex [**12**].

1. Statement of results.

Unless the contrary is stated, the manifold is assumed to be oriented from now on.

Let M be an m-dimensional manifold. Assume there is a map $f: M\rightarrow N$, where N is an aspherical manifold such that $f^{*}: H^{k}(N: Z)\rightarrow H^{k}(M: Z)$ is non-trivial for some integer $k\,(1\leqq k\leqq \dim M)$, where Z denotes the group of integers. We shall prove the following

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