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Nonexistence of bounded functions on the homology covering surface of $P^1-\{3 \text{ points}\}$

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Introduction.

Let D be the Riemann sphere punctured at three points. We consider the existence problem of bounded functions on its homology covering surface R, that is, the normal covering surface of D determined by the commutator subgroup of its fundamental group. We shall prove the following

THEOREM I. *The homology covering surface of the Riemann sphere punctured at three points belongs to the class* O_{AB}.

A complex manifold is said to belong to the *class* O_{AB} if and only if it carries no bounded, holomorphic, uniform and nonconstant function.

The motivation of this study rose from the question if the universal covering manifold of the complement of $n+2$ hyperplanes in general position in the n dimensional complex projective space $\boldsymbol{P}^n$ belongs to the class O_{AB}. By means of Theorem I, we solved this question affirmatively as follows.

THEOREM II ([7]). *If* $n \geqq 2$, *the universal covering manifold of the complement of* $n+2$ *hyperplanes in general position in* $\boldsymbol{P}^n$ *belongs to the class* O_{AB}.

In our previous paper [7] we announced Theorem I and derived Theorem II from Theorem I. The purpose of the present paper is to give the proof of Theorem I.

The proof of Theorem I is based on the criterion of A. Pfluger [5] which asserts that a Riemann surface having an exhaustion with some suitable properties belongs to the class O_{AB}.

The proof will be carried out as follows. In order to study topological and analytic properties of the homology covering surface R, we divide it into an infinite number of triangles. Then we place all these triangles on the complex plane properly, and glue them according to a certain rule, and thus we reconstruct the surface R, realizing it on the complex plane. This enables us to have a visual image of R and to treat it with ease.

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